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Full Paper

On $I^{(T)}\alpha$-open set and $I^{(T)}\beta$-open set in intuitionistic topological spaces

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Abstract: We introduce some new types of sets called intuitionistic semi-open set, $I^{(T)}\alpha$-open set, intuitionistic pre-open set and $I^{(T)}\beta$-open set in intuitionistic topological spaces. Also, we discuss some of their properties.

Keywords: intuitionistic semi-open set, $I^{(T)}\alpha$-open set, intuitionistic pre-open set, $I^{(T)}\beta$-open set

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INTRODUCTION

After Atanassov [1] introduced the concept of ‘intuitionistic fuzzy sets’ as a generalisation of fuzzy sets, it became a popular topic of investigation in the fuzzy set community. Many mathematical advantages of intuitionistic fuzzy sets have been discussed. Coker [2] generalised topological structures in fuzzy topological spaces to intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Later many researchers have studied topics related to intuitionistic fuzzy topological spaces.

On the other hand, Coker [3] introduced the concept of ‘intuitionistic sets’ in 1996. This is a discrete form of intuitionistic fuzzy set, where all the sets are entirely crisp sets. Still, it has membership and non-membership degrees, so this concept gives us more flexible approaches to representing vagueness in mathematical objects including those in engineering fields with classical set logic. In 2000 Coker [4] also introduced the concept of intuitionistic topological spaces with intuitionistic set and investigated the basic properties of continuous functions and compactness. He and his colleague [5, 6] also examined separation axioms in intuitionistic topological spaces.

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PRELIMINARIES

Definition 1 [3]. Let X be a non-empty fixed set. An intuitionistic set (IS) A is an object having the form $A = \langle X, A^1, A^2 \rangle$, where A^1 and A^2 are subsets of X satisfying $A^1 \cap A^2 = \phi$. The set A^1 is called the set of members of A, while A^2 is called the set of non-members of A.

Every crisp set A on a non-empty set X is obviously an IS having the form $\langle X, A, A^c \rangle$, and one can define several relations and operations between $IS's$ as follows:

Definition 2 [3]. Let X be a non-empty set, $A = \langle X, A^1, A^2 \rangle$ and $B = \langle X, B^1, B^2 \rangle$ be $IS's$ on X, and let $\{A_i : i \in J\}$ be an arbitrary family of $IS's$ in X, where $A^i = \langle X, A_i^1, A_i^2 \rangle$. Then

(a) $A \subseteq B$ if and only if $A^1 \subseteq B^1$ and $B^2 \subseteq A^2$;
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$;
(c) $A \subset B$ if and only if $A^1 \cup A^2 \supseteq B^1 \cup B^2$;
(d) $\bar{A} = \langle X, A^2, A^1 \rangle$;
(e) $\cup A^i = \langle X, \cup A_i^1, \cap A_i^2 \rangle$;
(f) $\cap A^i = \langle X, \cap A_i^1, \cup A_i^2 \rangle$;
(g) $A - B = A \cap \bar{B}$;
(h) $[\,]A = \langle X, A^1, (A^1)^c \rangle$;
(i) $\langle\ \rangle A = \langle X, (A^2)^c, A^2 \rangle$;
(j) $\phi_\sim = \langle X, \phi, X \rangle$ and $X_\sim = \langle X, X, \phi \rangle$.

The following are the basic properties of inclusion and complementation:

Corollary 1 [3]. Let A, B, C and A_i be $IS's$ in $(i \in J)$. Then

(a) $A_i \subseteq B$ for each $i \in J \Rightarrow \cup A_i \subseteq B$;
(b) $B \subseteq A_i$ for each $i \in J \Rightarrow B \subseteq \cap A_i$;
(c) $\overline{\cup A_i} = \cap \bar{A}_i$, $\overline{\cap A_i} = \cup \bar{A}_i$;
(d) $A \subseteq B \Leftrightarrow \bar{B} \subseteq \bar{A}$;
(e) $\overline{(\bar{A})} = A$;
(f) $\overline{\phi_\sim} = X_\sim$,$\overline{X_\sim} = \phi_\sim$.

Now we generalise the concept of 'intuitionistic fuzzy topological space' to intuitionistic sets:

Definition 3 [2]. An intuitionistic topology (IT) on a non-empty set X is a family T of $IS's$ in X satisfying the following axioms:

(i) $\phi_\sim, X_\sim \in T$,

(ii) $G_1 \cap G_2 \in T$ for any $G_1, G_2 \in T$,

(iii) $\cup G_i \in T$ for any arbitrary family $\{G_i : i \in J\} \subseteq T$.

In this case the pair (X, T) is called an intuitionistic topological space (ITS) and any IS in T is known as an intuitionistic open set (IOS) in X.

Example 1 [4]. Any topological space (X, T) is obviously an ITS in the form $T = \langle A', A \in T_0 \rangle$, whenever we identify a subset A in X with its counterpart $A' = \langle X, A, A^c \rangle$ as before.

Example 2 [4]. Let $X = \{a, b, c, d, e\}$ and consider the family $T = \{\phi_\sim, X_\sim, G_1, G_2, G_3, G_4\}$, where $G_1 = \langle X, \{a, b\}, \{d\} \rangle$, $G_2 = \langle X, \{a, c, e\}, \{b, d\} \rangle$, $G_3 = \langle X, \{a\}, \{b, d, e\} \rangle$ and $G_4 = \langle X, \{a, d\}, \{e\} \rangle$. Then (X, T) is an ITS on X.

Example 3 [4]. Let $X = \{a, b, c, d, e\}$ and consider the family $T = \{\emptyset_\sim, X_\sim, A_1, A_2, A_3, A_4\}$, where $A_1 = \langle X, \{a, b, c\}, \{d\} \rangle$, $A_2 = \langle X, \{c, d\}, \{e\} \rangle$, $A_3 = \langle X, \{c\}, \{d, e\} \rangle$ and $A_4 = \langle X, \{a, b, c, d\}, \emptyset \rangle$. Then (X, T) is an ITS on X.

Definition 4 [4]. The complement $\bar{A}$ of an intuitionistic open set (IOS) A in an ITS (X, T) is called an intuitionistic closed set (ICS) in X.

Now we define closure and interior operations in $ITS's$:

Definition 5 [7]. Let (X, T) be an ITS and $A = \langle X, A^1, A^2 \rangle$ be an IS in X. Then the interior and closure of A are defined by

$$int(A) = \cup\{K: K \text{ is an } IOS \text{ in } X \text{ and } K \subseteq A\},$$
$$cl(A) = \cap\{K: K \text{ is an } ICS \text{ in } X \text{ and } A \subseteq K\}.$$

In this paper we use $I^{(T)}c(A)$ instead of $cl(A)$ and $I^{(T)}i(A)$ instead of $int(A)$. It can also be shown that $I^{(T)}c(A)$ is an ICS and $I^{(T)}i(A)$ is an IOS in X, and A is an ICS in X if and only if $I^{(T)}c(A) = A$ and A is an IOS in X if and only if $I^{(T)}i(A) = A$.

Example 4 [4]. Let $X = \{a, b, c, d, e\}$ and consider the family $T = \{\phi_\sim, X_\sim, G_1, G_2, G_3, G_4\}$, where $G_1 = \langle X, \{a, b, c\}, \{d\}\rangle$, $G_2 = \langle X, \{c, d\}, \{e\}\rangle$, $G_3 = \langle X, \{c\}, \{d, e\}\rangle$ and $G_4 = \langle X, \{a, b, c, d\}, \{\phi\}\rangle$. If $B = \langle X, \{a, c\}, \{d\}\rangle$, then we can write $I^{(T)}i(B) = \langle X, \{c\}, \{d, e\}\rangle$ and $I^{(T)}c(B) = \langle X, X, \phi\rangle = X_\sim$.

Proposition 1 [4]. For any *IS* A in (X, T), we have $I^{(T)}c(\bar{A}) = \overline{I^{(T)}i(A)}$ and $I^{(T)}i(\bar{A}) = \overline{I^{(T)}c(A)}$.

Proposition 2 [4]. Let (X, T) be an ITS and A, B be $IS's$ in X. Then the following properties hold:

(a) $int(A) \subseteq A$; (b) $A \subseteq cl(A)$;
(c) $A \subseteq B \Rightarrow int(A) \subseteq int(B)$; (d) $A \subseteq B \Rightarrow cl(A) \subseteq cl(B)$;
(e) $int(int(A)) = int(A)$; (f) $cl(cl(A)) = cl(A)$;
(g) $int(A \cap B) = int(A) \cap int(B)$; (h) $cl(A \cup B) = cl(A) \cup cl(B)$;
(i) $int(X_\sim) = X_\sim$; (j) $cl(\phi_\sim) = \phi_\sim$.

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Definition 6. An *IS* $A = \langle X, A^1, A^2 \rangle$ in an $ITS(X, T)$ is said to be

(a) intuitionistic semi-open ($I^{(T)}S$-open) if $A \subseteq I^{(T)}c\left(I^{(T)}i(A)\right)$;

(b) intuitionistic α-open ($I^{(T)}\alpha$-open) if $A \subseteq I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A)\right)\right)$;

(c) intuitionistic pre-open ($I^{(T)}P$-open) if $A \subseteq I^{(T)}i\left(I^{(T)}c(A)\right)$.

The family of all $I^{(T)}S$-open ($I^{(T)}\alpha$-open, $I^{(T)}P$-open) sets of an $ITS(X, T)$ is denoted by $I^{(T)}SO(X)$ ($I^{(T)}\alpha O(X)$, $I^{(T)}PO(X)$).

An *IS* $A = \langle X, A^1, A^2 \rangle$ in an $ITS(X, T)$ is said to be $I^{(T)}S$-closed ($I^{(T)}\alpha$-closed, $I^{(T)}P$-closed) if $\bar{A} \in I^{(T)}SO(X)$ ($I^{(T)}\alpha O(X)$, $I^{(T)}PO(X)$).

Example 5. Let $X = \{a, b, c, d, e\}$ and consider the *IT* $T = \{\phi_\sim, X_\sim, G_1, G_2\}$, where $G_1 = \langle X, \{a, c\}, \{b, d, e\}\rangle$ and $G_2 = \langle X, \{a, c, e\}, \{b, d\}\rangle$. Let $A = \langle X, \{a, c, e\}, \{b\}\rangle$. Here A is an $I^{(T)}S$-open set since $I^{(T)}c\left(I^{(T)}i(\langle X, \{a, c, e\}, \{b\}\rangle)\right) = X_\sim$, $A \subseteq I^{(T)}c\left(I^{(T)}i(A)\right)$.

Example 6. Let $X = \{a, b, c, d\}$ and consider the *IT* $T = \{\phi_\sim, X_\sim, G_1, G_2\}$, where $G_1 = \langle X, \{a\}, \{b, c, d\}\rangle$ and $G_2 = \langle X, \{a, b\}, \{c, d\}\rangle$. Let $A = \langle X, \{a\}, \{b\}\rangle$. Here A is an $I^{(T)}\alpha$-open set since $I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(\langle X, \{a\}, \{b\}\rangle)\right)\right) = X_\sim$, $A \subseteq I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A)\right)\right)$.

Example 7. Let $X = \{a, b, c\}$ and consider the *IT* $T = \{\phi_\sim, X_\sim, G, H\}$, where $G = \langle X, \{a\}, \{b, c\}\rangle$ and $H = \langle X, \{a, b\}, \{c\}\rangle$. Let $A = \langle X, \{b\}, \{c\}\rangle$. Here A is an $I^{(T)}P$-open set since $I^{(T)}i\left(I^{(T)}c(\langle X, \{b\}, \{c\}\rangle)\right) = X_\sim$, $A \subseteq I^{(T)}i\left(I^{(T)}c(A)\right)$.

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Theorem 1. (a) Every $I^{(T)}$-open set is $I^{(T)}\alpha$-open set; (b) every $I^{(T)}\alpha$-open set is $I^{(T)}P$-open set.

Proof. (a) Let A be an IOS in (X, T). Since $A \subseteq I^{(T)}c(A)$, then $I^{(T)}i(A) \subseteq I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A)\right)\right)$. Hence $A \subseteq I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A)\right)\right)$ and A is an $I^{(T)}\alpha$-open set.

(b) Let A be $I^{(T)}\alpha$ -open set in (X, T) . Then it follows that $A \subseteq I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A)\right)\right) \subseteq I^{(T)}i\left(I^{(T)}c(A)\right)$. Therefore, A is an $I^{(T)}P$-open set.

Remark 1. The separate converses need not be true in general, which is shown by the following examples.

Example 8. Let $X = \{a, b, c, d\}$ and consider the IT $T = \{\phi_\sim, X_\sim, G_1, G_2\}$, where $G_1 = \langle X, \{a\}, \{b, c, d\}\rangle$ and $G_2 = \langle X, \{a, b\}, \{c, d\}\rangle$. Let $A = \langle X, \{a\}, \{b\}\rangle$. Here A is an $I^{(T)}\alpha$-open set but not an IOS since $I^{(T)}i(A) = \langle X, \{a\}, \{b, c, d\}\rangle \neq A$.

Example 9. Let $X = \{a, b, c\}$ and consider the IT $T = \{\phi_\sim, X_\sim, G, H\}$, where $G = \langle X, \{a\}, \{b, c\}\rangle$ and $H = \langle X, \{a, b\}, \{c\}\rangle$. Let $A = \langle X, \{b\}, \{c\}\rangle$. Here A is an $I^{(T)}P$-open set but not an $I^{(T)}\alpha$-open set since A is not contained in $I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A)\right)\right)$.

Theorem 2. The finite union of $I^{(T)}\alpha$-open set is always an $I^{(T)}\alpha$-open set.

Proof. Let A and B be two $I^{(T)}\alpha$ -open sets. Then $A \subseteq I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A)\right)\right)$ and $B \subseteq I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(B)\right)\right)$ imply that $A \cup B \subseteq I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A \cup B)\right)\right)$. Therefore, $A \cup B$ is an $I^{(T)}\alpha$-open set.

Proposition 3. Let (X, T) be an ITS and let $A \in IS(X)$. Then $A \in I^{(T)}PO(X) \Leftrightarrow (\exists B \in T)\left(A \subseteq B \subseteq I^{(T)}c(A)\right)$.

Proof. If $A \in I^{(T)}PO(X)$, then $A \subseteq I^{(T)}i\left(I^{(T)}c(A)\right)$. Take $B = I^{(T)}i\left(I^{(T)}c(A)\right)$. Then $B \in T$ and $A \subseteq B \subseteq I^{(T)}c(A)$.

Conversely, let $B \in T$ be such that $A \subseteq B \subseteq I^{(T)}c(A)$. Then $A \subseteq I^{(T)}i(B) \subseteq I^{(T)}i\left(I^{(T)}c(A)\right)$ and so $A \in I^{(T)}PO(X)$.

Theorem 3. Let (X, T) be an ITS. A sub-set A of X is an $I^{(T)}\alpha$-open set if and only if it is $I^{(T)}S$-open set and $I^{(T)}P$-open set.

Proof. *Necessity:* Let A be $I^{(T)}\alpha$-open set. Then we have $A \subseteq I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A)\right)\right)$. This implies that $A \subseteq I^{(T)}c\left(I^{(T)}i(A)\right)$ and $A \subseteq I^{(T)}i\left(I^{(T)}c(A)\right)$. Hence A is $I^{(T)}S$-open set and $I^{(T)}P$-open set.

Sufficiency: Let A be $I^{(T)}S$-open set and $I^{(T)}P$-open set. Then we have $A \subseteq I^{(T)}i\left(I^{(T)}c(A)\right) \subseteq I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A)\right)\right)$. This shows that A is an $I^{(T)}\alpha$-open set.

Definition 7. A sub-set A of X is said to be an $I^{(T)}\alpha$-closed set if and only if $X - A$ is an $I^{(T)}\alpha$-open set, which is equivalent.

Let (X, T) be an ITS and A be a sub-set X. Then A is an $I^{(T)}\alpha$-closed set if and only if $A \supseteq I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A)\right)\right)$.

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Definition 8. Let (X, T) be an *ITS* and A be a sub-set X. Then

(a) $I^{(T)}\alpha$-interior of A is the union of all $I^{(T)}\alpha$-open sets contained in A and it is denoted by $Int_{I^{(T)}}\alpha(A)$.

$Int_{I^{(T)}}\alpha(A) = \cup\{U: U \text{ is an } I^{(T)}\alpha - \text{open set and } U \subseteq A\}$.

(b) $I^{(T)}\alpha$-closure of A is the intersection of all $I^{(T)}\alpha$-closed sets containing A and it is denoted by $Cl_{I^{(T)}}\alpha(A)$.

$Cl_{I^{(T)}}\alpha(A) = \cap\{U: U \text{ is an } I^{(T)}\alpha - \text{closed set and } A \subseteq U\}$.

(c) $I^{(T)}P$-interior of A is the union of all $I^{(T)}$-preopen sets contained in A and it is denoted by $Int_{I^{(T)}}P(A)$.

$Int_{I^{(T)}}P(A) = \cup\{U: U \text{ is an } I^{(T)} - \text{preopen set and } U \subseteq A\}$.

(d) $I^{(T)}P$-closure of A is the intersection of all $I^{(T)}$-preclosed sets containing A and it is denoted by $Cl_{I^{(T)}}P(A)$.

$Cl_{I^{(T)}}P(A) = \cap\{U: U \text{ is an } I^{(T)} - \text{preclosed set and } A \subseteq U\}$.

(e) $I^{(T)}S$-interior of A is the union of all $I^{(T)}$-semiopen sets contained in A and it is denoted by $Int_{I^{(T)}}S(A)$.

$Int_{I^{(T)}}S(A) = \cup\{U: U \text{ is an } I^{(T)} - \text{semiopen set and } U \subseteq A\}$.

(f) $I^{(T)}S$-closure of A is the intersection of all $I^{(T)}$-semiclosed sets containing A and it is denoted by $Cl_{I^{(T)}}S(A)$.

$Cl_{I^{(T)}}S(A) = \cap\{U: U \text{ is an } I^{(T)} - \text{semiclosed set and } A \subseteq U\}$.

Remark 2. It is clear that $Int_{I^{(T)}}\alpha(A)$ is an $I^{(T)}\alpha$-open set and $Cl_{I^{(T)}}\alpha(A)$ is an $I^{(T)}\alpha$-closed set.

Theorem 4. An *IS* $A = \langle X, A^1, A^2\rangle$ in an $ITS(X, T)$. Then

(a) $X - Int_{I^{(T)}}\alpha(A) = Cl_{I^{(T)}}\alpha(X - A)$.

(b) $X - Cl_{I^{(T)}}\alpha(A) = Int_{I^{(T)}}\alpha(X - A)$.

Proof. (a) and (b) are clear.

Observation 1. The following statements are true for every A and B:

(a) $Int_{I^{(T)}}\alpha(A) \cup Int_{I^{(T)}}\alpha(B) = Int_{I^{(T)}}\alpha(A \cup B)$.

(b) $Cl_{I^{(T)}}\alpha(A) \cap Cl_{I^{(T)}}\alpha(B) = Cl_{I^{(T)}}\alpha(A \cap B)$.

Proof. Obvious.

Observation 2. Let A be a sub-set of a space (X, T). Then

(a) $Cl_{I^{(T)}}\alpha(A) = A \cup I^{(T)}c\left(I^{(T)}i\left(I^{(T)}c(A)\right)\right)$.

(b) $Int_{I^{(T)}}\alpha(A) = A \cap I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A)\right)\right)$.

(c) $Cl_{I^{(T)}}S(A) = A \cup I^{(T)}i\left(I^{(T)}c(A)\right)$.

(d) $Int_{I^{(T)}}S(A) = A \cap I^{(T)}c\left(I^{(T)}i(A)\right)$.

(e) $Cl_{I^{(T)}}P(A) = A \cup I^{(T)}c\left(I^{(T)}i(A)\right)$.

(f) $Int_{I^{(T)}}P(A) = A \cap I^{(T)}i\left(I^{(T)}c(A)\right)$.

Proof. Obvious.

Example 10. Let $X = \{a, b, c, d, e\}$ and consider the *IT* $T = \{\phi_\sim, X_\sim, G_1, G_2\}$, where $G_1 = \langle X, \{a, c\}, \{b, d, e\}\rangle$ and $G_2 = \langle X, \{a, c, e\}, \{b, d\}\rangle$. Let $A = \langle X, \{a, c, e\}, \{b\}\rangle$. Then

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(a) $Cl_{I^{(T)}}\alpha(A) = \langle X, \{a, c, e\}, \{b\}\rangle \cup I^{(T)}c\left(I^{(T)}i\left(I^{(T)}c(\langle X, \{a, c, e\}, \{b\}\rangle)\right)\right)$

$= \langle X, \{a, c, e\}, \{b\}\rangle \cup X_{\sim} = X_{\sim}$.

(b) $Int_{I^{(T)}}\alpha(A) = \langle X, \{a, c, e\}, \{b\}\rangle \cap I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(\langle X, \{a, c, e\}, \{b\}\rangle)\right)\right)$

$= \langle X, \{a, c, e\}, \{b\}\rangle \cap X_{\sim} = A$.

(c) $Cl_{I^{(T)}}S(A) = \langle X, \{a, c, e\}, \{b\}\rangle \cup I^{(T)}i\left(I^{(T)}c(\langle X, \{a, c, e\}, \{b\}\rangle)\right)$

$= \langle X, \{a, c, e\}, \{b\}\rangle \cup X_{\sim} = X_{\sim}$.

(d) $Int_{I^{(T)}}S(A) = \langle X, \{a, c, e\}, \{b\}\rangle \cap I^{(T)}c\left(I^{(T)}i(\langle X, \{a, c, e\}, \{b\}\rangle)\right)$

$= \langle X, \{a, c, e\}, \{b\}\rangle \cap X_{\sim} = A$.

(e) $Cl_{I^{(T)}}P(A) = \langle X, \{a, c, e\}, \{b\}\rangle \cup I^{(T)}c\left(I^{(T)}i(\langle X, \{a, c, e\}, \{b\}\rangle)\right)$

$= \langle X, \{a, c, e\}, \{b\}\rangle \cup X_{\sim} = X_{\sim}$.

(f) $Int_{I^{(T)}}P(A) = \langle X, \{a, c, e\}, \{b\}\rangle \cap I^{(T)}i\left(I^{(T)}c(\langle X, \{a, c, e\}, \{b\}\rangle)\right)$

$= \langle X, \{a, c, e\}, \{b\}\rangle \cap X_{\sim} = A$.

Observation 3. Let A be a sub-set of a space (X, T). Then

(a) $Cl_{I^{(T)}}S\left(Int_{I^{(T)}}S(A)\right) = Int_{I^{(T)}}S(A) \cup I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A)\right)\right)$.

(b) $Cl_{I^{(T)}}P\left(Int_{I^{(T)}}P(A)\right) = Int_{I^{(T)}}P(A) \cup I^{(T)}c\left(I^{(T)}i(A)\right)$.

(c) $I^{(T)}i\left(Cl_{I^{(T)}}S(A)\right) = Int_{I^{(T)}}P\left(I^{(T)}c(A)\right) = Int_{I^{(T)}}P\left(Cl_{I^{(T)}}S(A)\right)$

$= Cl_{I^{(T)}}S\left(Int_{I^{(T)}}P(A)\right) = I^{(T)}i\left(I^{(T)}c(A)\right)$.

Example 11. Let $X = \{l, m, n, o, p, q\}$ and consider the IT $T = \{\phi_{\sim}, X_{\sim}, L, M\}$, where $L = \langle X, \{l, n, p\}, \{m, o, q\}\rangle$ and $M = \langle X, \{l, n\}, \{m, q\}\rangle$. Let $A = \langle X, \{l, n\}, \{m\}\rangle$. Then

(a) $Cl_{I^{(T)}}S\left(Int_{I^{(T)}}S(A)\right) = Int_{I^{(T)}}S(A) \cup I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A)\right)\right)$.

$I^{(T)}i(A) = \langle X, \{l, n\}, \{m\}\rangle$,

$I^{(T)}c\left(I^{(T)}i(A)\right) = I^{(T)}c(\langle X, \{l, n\}, \{m\}\rangle) = X_{\sim}$, $I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A)\right)\right) = X_{\sim}$,

$Int_{I^{(T)}}S(A) = A$,

$Int_{I^{(T)}}S(A) \cup I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A)\right)\right) = A \cup X_{\sim} = X_{\sim}$ (1)

$Cl_{I^{(T)}}S\left(Int_{I^{(T)}}S(A)\right) = Cl_{I^{(T)}}S(A) = X_{\sim}$ (2)

From (1) and (2), $Cl_{I^{(T)}}S\left(Int_{I^{(T)}}S(A)\right) = Int_{I^{(T)}}S(A) \cup I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A)\right)\right)$.

(b) $Cl_{I^{(T)}}P\left(Int_{I^{(T)}}P(A)\right) = Int_{I^{(T)}}P(A) \cup I^{(T)}c\left(I^{(T)}i(A)\right)$

$Int_{I^{(T)}}P(A) = A \cap I^{(T)}i\left(I^{(T)}c(A)\right) = \langle X, \{l, n\}, \{m\}\rangle \cap I^{(T)}i\left(I^{(T)}c(\langle X, \{l, n\}, \{m\}\rangle)\right) = A$,

$I^{(T)}c\left(I^{(T)}i(A)\right) = X_{\sim}$, $Int_{I^{(T)}}P(A) \cup I^{(T)}c\left(I^{(T)}i(A)\right) = A \cup X_{\sim} = X_{\sim}$ (3)

$Cl_{I^{(T)}}P\left(Int_{I^{(T)}}P(A)\right) = Cl_{I^{(T)}}P(A) = \langle X, \{l, n\}, \{m\}\rangle \cup I^{(T)}c\left(I^{(T)}i(\langle X, \{l, n\}, \{m\}\rangle)\right) = X_{\sim}$ (4)

From (3) and (4), $Cl_{I^{(T)}}P\left(Int_{I^{(T)}}P(A)\right) = Int_{I^{(T)}}P(A) \cup I^{(T)}c\left(I^{(T)}i(A)\right)$.

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(c)
$I^{(T)}i\left(Cl_{I^{(T)}}S(A)\right) = Int_{I^{(T)}}P\left(I^{(T)}c(A)\right) = Int_{I^{(T)}}P\left(Cl_{I^{(T)}}S(A)\right) = Cl_{I^{(T)}}S\left(Int_{I^{(T)}}P(A)\right) = I^{(T)}i\left(I^{(T)}c(A)\right)$

$$Cl_{I^{(T)}}S(A) = X_\sim,\ I^{(T)}i\left(Cl_{I^{(T)}}S(A)\right) = I^{(T)}i(X_\sim) = X_\sim \quad (5)$$

$$I^{(T)}c(A) = X_\sim,\ Int_{I^{(T)}}P\left(I^{(T)}c(A)\right) = Int_{I^{(T)}}P(X_\sim) = X_\sim \quad (6)$$

$$Cl_{I^{(T)}}S(A) = X_\sim,\ Int_{I^{(T)}}P\left(Cl_{I^{(T)}}S(A)\right) = Int_{I^{(T)}}P(X_\sim) = X_\sim \quad (7)$$

$$Int_{I^{(T)}}P(A) = A,\ Cl_{I^{(T)}}S\left(Int_{I^{(T)}}P(A)\right) = Cl_{I^{(T)}}S(A) = X_\sim \quad (8)$$

$$I^{(T)}c(A) = X_\sim,\ I^{(T)}i\left(I^{(T)}c(A)\right) = I^{(T)}i(X_\sim) = X_\sim \quad (9)$$

From (5), (6), (7), (8) and (9), $I^{(T)}i\left(Cl_{I^{(T)}}S(A)\right) = Int_{I^{(T)}}P\left(I^{(T)}c(A)\right) = Int_{I^{(T)}}P\left(Cl_{I^{(T)}}S(A)\right) = Cl_{I^{(T)}}S\left(Int_{I^{(T)}}P(A)\right) = I^{(T)}i\left(I^{(T)}c(A)\right)$.

Definition 9. An *IS* $A = \langle X, A^1, A^2\rangle$ in an $ITS(X,\ T)$ is said to be
(a) intuitionistic semi-preopen ($I^{(T)}SP$-open or $I^{(T)}\beta$-open) if there exists $B \in I^{(T)}PO(X)$ such that $B \subseteq A \subseteq I^{(T)}c(A)$. The family of all $I^{(T)}\beta$-open sets of an $ITS(X,\ T)$ will be denoted by $I^{(T)}\beta O(X)$.
(b) intuitionistic semi-preclosed ($I^{(T)}SP$-closed or $I^{(T)}\beta$-closed) if there exists an intuitionistic preclosed set B such that $I^{(T)}i(A) \subseteq A \subseteq B$. The family of all $I^{(T)}\beta$-closed sets of an $ITS(X,\ T)$ will be denoted by $I^{(T)}\beta C(X)$.

Example 12. Let $X = \{a,\ b,\ c\}$ and $G_1 = \langle X,\ \{a\},\ \{b, c\}\rangle$, $G_2 = \langle X,\ \{a,\ b\},\ \{c\}\rangle$, $G_3 = \langle X,\ \{a\},\ \{b\}\rangle$. Then $T = \{\phi_\sim, X_\sim, G_1,\ G_2\}$ is an *IT* on X and $G_3 \in I^{(T)}PO(X)$.

Let $A = \langle X,\ \{a,\ c\},\ \{b\}\rangle$ be an IS in $(X,\ T)$. Then $G_3 \subseteq A \subseteq I^{(T)}c(G_3)$, and hence $A \in I^{(T)}\beta O(X)$.

Theorem 5. For every $ISA = \langle X, A^1, A^2\rangle$ in $(X,\ T)$, we have $A \in I^{(T)}\beta C(X) \Leftrightarrow \bar{A} \in I^{(T)}\beta O(X)$.
Proof. Straightforward.

Theorem 6. Every $I^{(T)}S$-open set is an $I^{(T)}\beta$-open set.
Proof. Let A be $I^{(T)}S$-open set in $(X,\ T)$. Then it follows that $A \subseteq I^{(T)}c\left(I^{(T)}i(A)\right) \subseteq I^{(T)}c\left(I^{(T)}i\left(I^{(T)}c(A)\right)\right)$. Hence A is an $I^{(T)}\beta$-open set.

Remark 3. The converse of the above theorem need not be true in general. It is shown by the following example.

Example 13. Let $X = \{a,\ b,\ c\}$ and consider the *IT* $T = \{\phi_\sim,\ X_\sim,\ P,\ Q\}$, where $P = \langle X,\ \{a\},\ \{b, c\}\rangle$ and $Q = \langle X,\ \{a,\ b\},\ \{c\}\rangle$. Let $A = \langle X,\ \{b\},\ \{c\}\rangle$. Here A is an $I^{(T)}\beta$-open set but not an $I^{(T)}S$-open set since A is not contained in $I^{(T)}c\left(I^{(T)}i(A)\right)$.

Theorem 7. For every $ISA = \langle X, A^1, A^2\rangle$ in $(X,\ T)$, we have $A \in I^{(T)}\beta O(X) \Leftrightarrow \bar{A} \in I^{(T)}\beta C(X)$.
Proof. Straightforward.

Theorem 8. Let $(X,\ T)$ be an ITS. Then
(a) Any union of $I^{(T)}\beta$-open sets is an $I^{(T)}\beta$-open set.
(b) Any intersection of $I^{(T)}\beta$-closed sets is an $I^{(T)}\beta$-closed set.

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Proof. (a) Let $\{A_i\}_{i\in J}$ be a collection of $I^{(T)}\beta$-open sets of $(X,\ T)$. Then there exists $B_i \in I^{(T)}PO(X)$ such that $B_i \subseteq A_i \subseteq I^{(T)}c(B_i)$ for each $i \in J$. It follows that $\cup B_i \subseteq \cup A_i \subseteq \cup I^{(T)}c(\cup B_i)$ and $\cup B_i \in I^{(T)}PO(X)$. Hence $\cup A_i \in I^{(T)}\beta O(X)$.
(b) This is from (a) by taking compliments.

Theorem 9. For any *IS* $A = \langle X, A^1, A^2\rangle$ in an $ITS(X,\ T)$, $A \in I^{(T)}\beta O(X)$ if and only if $(\forall p(\alpha,\ \beta) \in A)\left(\exists B \in I^{(T)}\beta O(X)\right)(p(\alpha,\ \beta) \in B \subseteq A)$.

Proof. If $A \in I^{(T)}\beta O(X)$, then we can take $B = A$ so that $p(\alpha,\ \beta) \in B \subseteq A$ for every $p(\alpha,\ \beta) \in A$. Let A be an *IS* in $(X,\ T)$ and assume that there exists $B \in I^{(T)}\beta O(X)$ such that $p(\alpha,\ \beta) \in B \subseteq A$. Then $A = \cup_{p(\alpha,\beta)\in A}\{p(\alpha,\ \beta)\} = \cup_{p(\alpha,\beta)\in A} B \subseteq A$, and so $A = \cup_{p(\alpha,\beta)\in A} B$, which is an $I^{(T)}\beta$-open set by Theorem 8(a).

Theorem 10. Let $(X,\ T)$ be an *ITS*. Then
(a) $\left(\forall A \in I^{(T)}\beta O(X)\right)\left(\forall B \in I^{(T)}SO(X)\right)\left(A \subseteq B \subseteq I^{(T)}c(A)\right) \Rightarrow \forall B \in I^{(T)}\beta O(X)$.
(b) $\left(\forall A \in I^{(T)}\beta C(X)\right)\left(\forall B \in I^{(T)}SC(X)\right)\left(I^{(T)}i(A) \subseteq B \subseteq A\right) \Rightarrow \forall B \in I^{(T)}\beta C(X)$.

Proof. (a) Assume that $A \subseteq B \subseteq I^{(T)}c(A)$ for every $A \in I^{(T)}\beta O(X)$ and $B \in I^{(T)}SO(X)$. Let $C \in I^{(T)}PO(X)$ be such that $C \subseteq A \subseteq I^{(T)}c(C)$. Obviously, $C \subseteq B$. From $A \subseteq I^{(T)}c(C)$, it follows that $I^{(T)}c(A) \subseteq I^{(T)}c(C)$ so that $C \subseteq B \subseteq I^{(T)}c(A) \subseteq I^{(T)}c(C)$. Hence $B \in I^{(T)}\beta O(X)$.
(b) This follows from (a).

Definition 10. Let $(X,\ T)$ be an *ITS* and A be a sub-set X. Then
(a) $I^{(T)}\beta$-interior of A is the union of all $I^{(T)}\beta$-open sets contained in A and it is denoted by $Int_{I^{(T)}}\beta(A)$.

$Int_{I^{(T)}}\beta(A) = \cup\{U\colon\ U \text{ is an } I^{(T)}\beta - \text{open set and } U \subseteq A\}$.

(b) $I^{(T)}\beta$-closure of A is the intersection of all $I^{(T)}\beta$-closed sets containing A and it is denoted by $Cl_{I^{(T)}}\beta(A)$.

$Cl_{I^{(T)}}\beta(A) = \cap\{U\colon\ U \text{ is an } I^{(T)}\beta - \text{closed set and } A \subseteq U\}$.

Observation 4. Let A be a sub-set of a space $(X,\ T)$. Then
(a) $Cl_{I^{(T)}}\beta(A) = A \cup I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A)\right)\right)$.
(b) $Int_{I^{(T)}}\beta(A) = A \cap I^{(T)}c\left(I^{(T)}i\left(I^{(T)}c(A)\right)\right)$.

Example 14. Let $X = \{a,\ b,\ c,\ d,\ e\}$ and consider the *IT* $T = \{\phi_\sim, X_\sim, G_1, G_2\}$, where $G_1 = \langle X,\ \{a,\ c\},\ \{b, d, e\}\rangle$ and $G_2 = \langle X,\ \{a,\ c,\ e\},\ \{b,\ d\}\rangle$. Let $A = \langle X,\ \{a,\ c,\ e\},\ \{b\}\rangle$. Then
(a) $Cl_{I^{(T)}}\beta(\langle X,\ \{a,\ c,\ e\},\ \{b\}\rangle) = \langle X,\ \{a,\ c,\ e\},\ \{b\}\rangle \cup I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(\langle X,\ \{a,\ c,\ e\},\ \{b\}\rangle)\right)\right) = X_\sim$.
(b) $Int_{I^{(T)}}\beta(\langle X,\ \{a,\ c,\ e\},\ \{b\}\rangle) = \langle X,\ \{a,\ c,\ e\},\ \{b\}\rangle \cap I^{(T)}c\left(I^{(T)}i\left(I^{(T)}c(\langle X,\ \{a,\ c,\ e\},\ \{b\}\rangle)\right)\right) = A$.

Observation 5. Let A be a sub-set of a space $(X,\ T)$. Then
(a) $Cl_{I^{(T)}}\beta\left(Int_{I^{(T)}}\beta(A)\right) = Int_{I^{(T)}}\beta\left(Cl_{I^{(T)}}\beta(A)\right)$.
(b) $I^{(T)}i\left(Cl_{I^{(T)}}P(A)\right) = Cl_{I^{(T)}}S\left(I^{(T)}i(A)\right) = Cl_{I^{(T)}}\beta\left(I^{(T)}i(A)\right) = I^{(T)}i\left(Cl_{I^{(T)}}\beta(A)\right)$
$= I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A)\right)\right)$.

Example 15. Let $X = \{l, m, n, o, p, q\}$ and consider the IT $T = \{\phi_\sim, X_\sim, L, M\}$, where $L = \langle X, \{l, n, p\}, \{m, o, q\}\rangle$ and $M = \langle X, \{l, n\}, \{m, q\}\rangle$. Let $A = \langle X, \{l, n\}, \{m\}\rangle$. Then

(a) $Cl_{I^{(T)}}\beta\left(Int_{I^{(T)}}\beta(A)\right) = Int_{I^{(T)}}\beta\left(Cl_{I^{(T)}}\beta(A)\right)$.

$Int_{I^{(T)}}\beta(\langle X, \{l, n\}, \{m\}\rangle) = \langle X, \{l, n\}, \{m\}\rangle \cap I^{(T)}c\left(I^{(T)}i\left(I^{(T)}c(\langle X, \{l, n\}, \{m\}\rangle)\right)\right) = A.$

$$Cl_{I^{(T)}}\beta\left(Int_{I^{(T)}}\beta(A)\right) = Cl_{I^{(T)}}\beta(A) = A \cup I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A)\right)\right) = A \cup X_\sim = X_\sim \quad (10)$$

$$Cl_{I^{(T)}}\beta(A) = X_\sim, Int_{I^{(T)}}\beta\left(Cl_{I^{(T)}}\beta(A)\right) = Int_{I^{(T)}}\beta(X_\sim) = X_\sim \quad (11)$$

From (10) and (11), $Cl_{I^{(T)}}\beta\left(Int_{I^{(T)}}\beta(A)\right) = Int_{I^{(T)}}\beta\left(Cl_{I^{(T)}}\beta(A)\right)$.

(b) $I^{(T)}i\left(Cl_{I^{(T)}}P(A)\right) = Cl_{I^{(T)}}S\left(I^{(T)}i(A)\right) = Cl_{I^{(T)}}\beta\left(I^{(T)}i(A)\right) = I^{(T)}i\left(Cl_{I^{(T)}}\beta(A)\right)$
$= I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A)\right)\right)$.

$$Cl_{I^{(T)}}P(A) = X_\sim, I^{(T)}i(X_\sim) = X_\sim \quad (12)$$

$$I^{(T)}i(A) = \langle X, \{l, n\}, \{m, q\}\rangle, Cl_{I^{(T)}}S\left(I^{(T)}i(A)\right) = Cl_{I^{(T)}}S(\langle X, \{l, n\}, \{m, q\}\rangle) = X_\sim \quad (13)$$

$$Cl_{I^{(T)}}\beta\left(I^{(T)}i(A)\right) = Cl_{I^{(T)}}\beta(\langle X, \{l, n\}, \{m, q\}\rangle) = X_\sim \quad (14)$$

$$Cl_{I^{(T)}}\beta(A) = Cl_{I^{(T)}}\beta(\langle X, \{l, n\}, \{m\}\rangle) = X_\sim, I^{(T)}i\left(Cl_{I^{(T)}}\beta(A)\right) = I^{(T)}i(X_\sim) = X_\sim \quad (15)$$

$$I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A)\right)\right) = X_\sim \quad (16)$$

From (12), (13), (14), (15) and (16),
$I^{(T)}i\left(Cl_{I^{(T)}}P(A)\right) = Cl_{I^{(T)}}S\left(I^{(T)}i(A)\right) = Cl_{I^{(T)}}\beta\left(I^{(T)}i(A)\right) = I^{(T)}i\left(Cl_{I^{(T)}}\beta(A)\right)$
$= I^{(T)}i\left(I^{(T)}c\left(I^{(T)}i(A)\right)\right)$.

CONCLUSIONS

We have introduced the sets in intuitionistic topological spaces called intuitionistic semi-open set, intuitionistic α-open set, intuitionistic pre-open set, $I^{(T)}\alpha$-interior of A, $I^{(T)}\alpha$-closure of A, $I^{(T)}P$-interior of A, $I^{(T)}P$-closure of A, $I^{(T)}S$-interior of A, $I^{(T)}S$-closure of A, intuitionistic semi-preopen set, intuitionistic semi-preclosed set, $I^{(T)}\beta$-interior of A and $I^{(T)}\beta$-closure of A, and studied some of their properties.

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