Maejo Int. J. Sci. Technol. **2016**, *10*(02), 233-241

Maejo International Journal of Science and Technology

ISSN 1905-7873
Available online at www.mijst.mju.ac.th

Full Paper

Multiplicative multi-criteria analysis method for decision-making

Miodrag M. Zizovic [1], Nada Damljanovic [2,*] and Malisa R. Zizovic [2,3]

[1] Faculty of Tourism and Hospitality Management, Singidunum University, Belgrade, Serbia
[2] Faculty of Technical Sciences in Cacak, University of Kragujevac, Serbia
[3] Faculty of Business Valjevo, Singidunum University, Belgrade, Serbia
* Corresponding author, e-mail: nada.damljanovic@gmail.com

Received: 3 March 2015 / Accepted: 12 August 2016 / Published: 18 August 2016

Abstract: In this paper we introduce a new method of multi-criteria analysis: the multiplicative method. We present some advantages of this method over the other commonly used multi-criteria methods such as the low-calculation complexity, the prevention of favouritism of alternatives and the cutting out of alternatives which do not satisfy an absolutely important criterion. We also present an example of the application of this method in a concrete situation: the choice of a brand name for Gornja Trepca Spa advertising.

Keywords: multiplicative method, multi-criteria analysis, degree of criteria

---

INTRODUCTION

We handle many multi-criteria problems of various importance everyday, most of which are resolved intuitively. Making a decision implies that there are alternative choices to be considered. For a given set of alternatives and a set of decision criteria, it is only natural to ask what the best alternative is. Scientifically, the multi-criteria decision problem is reduced to the task of comparison among a number of alternatives evaluated by a great number of different criteria (most often with different relative importance for making the decision).

The first difficulty we face when using different criteria for making a decision is the domain of values that each criterion can take. Some criteria take qualitative values (described subjectively) while others take quantitative ones (measured numerically). For instance, the price of a car is numerical and the comfort rating is qualitative.

The main task of an analyst is to understand the preference of the decision-maker and to develop a model that describes this preference. In a multi-criteria value model the decision-maker's preference in relation to a set of alternatives is given by a utility function which is the result of aggregation of value functions assigned to each criterion. The most commonly used aggregation function is the weighted sum, which is attractive due to its low complexity, but other aggregation functions can also be applied [1]. For instance, very popular are the non-additive approaches, where the aggregation function is not a linear combination of partial preferences (such as Chouqet integral [2]). Several methods involve complex calculations, so finding an optimal value can be a lengthy process. When making a choice, there is a tendency to make the decision-making process as efficient as possible. Selecting a method of optimisation depends on the type of problem to be solved, the knowledge and experience of the decision-maker in the field of multi-criteria analysis, as well as the technology issues taken into consideration [3, 4].

During the previous decades, searching for an optimal decision (solution) led to many decision methods and techniques that were proposed and elaborated within the scientific disciplines such as operations research, management science, computer science and statistics [e.g. 5-11]. In combination with the use of modern computers many of these methods have had an extensive software support. Multi-criteria decision analysis has been used in a wide variety of fields such as energy management, environmental planning, public services, healthcare, transportation, logistics, marketing, human resources management and finance [e.g. 12-24]. Multi-criteria decision analysis approaches have been widely used by public entities, firms and organisations [25]. For an overview of the available methods for solving multi-criteria decision problems, we refer to Figueira et al. [26], Hwang and Yoon [1], Radojicic and Zizovic [27], Triantaphyllou [28] and Zeleny [29].

For most multi-criteria decision methods it is common that introducing new alternatives into a model can change the rank of starting alternatives; see for example Zizovic and Damljanovic [30]. The lattice procedure introduced by Zizovic et al. [3] offers a possibility to avoid this situation, but this method is complex. In this paper we introduce a new method for multi-criteria decision analysis, namely the multiplicative multi-criteria decision method, which is based on the multiplicative aggregation function. This method is efficient and conducive to applications and its main advantage over most other methods is that it stabilises the decision process so that newly added alternatives do not change the order of starting alternatives.

MULTIPLICATIVE METHOD

In all practical problems associated with the selection and assessment, the number of alternatives is limited. Therefore, our focus concentrates on the problems with a finite number of alternatives. In this case a multi-criteria decision problem may be described using a decision matrix given in Table 1.

Suppose there are m alternatives, $A_1, A_2, \ldots, A_m$, to be assessed based on n maximisation criteria, $K_1, K_2, \ldots, K_n$. A decision matrix is an $m \times n$-matrix with each element a_{ij} being the j-th criterion value of the i-th alternative; that is, a_{ij} is the degree in which alternative A_i satisfies criterion K_j ($0 \le a_{ij} \le 1$). Notice that we assume that all criteria are of the maximisation type (because all criteria of the minimisation type can be transformed into the maximisation type).

In the multiplicative multi-criteria method each criterion K_j is associated with a degree of importance ρ_j $(0 < \rho_j \le 1)$ of the decision. Here we assume that all the criteria are arranged in strictly descending order, in the sense that the first criterion has the greatest importance and each following criterion has less importance for the decision than the previous one.

Maejo Int. J. Sci. Technol. **2016**, *10*(02), 233-241

Table 1. Decision matrix

	K_1	K_2	$\cdots$	K_n
A_1	a_{11}	a_{12}	$\cdots$	a_{1n}
A_2	a_{21}	a_{22}	$\cdots$	a_{2n}
$\vdots$	$\vdots$	$\vdots$		$\vdots$
A_m	a_{m1}	a_{m2}		a_{mn}

Given a decision matrix for a particular multi-criteria problem, it is naturally assumed that all alternatives are usually efficient, there being no alternative dominated by any other. When an alternative is better according to one criterion, the other is better according to the other criterion. Therefore, incomparability holds for all pair-wise comparisons.

To decide which alternative is the best solution, it is necessary to have some additional piece of information on the preference relation of the decision-maker. For example, it can be a reference point or minimal suitable value. To develop a multiplicative model that describes the preference relation, we use one hypothetical alternative, $A(a_1, a_2, \ldots, a_n)$, where $a_1, a_2, \ldots, a_n$ are degrees in which hypothetical alternative A satisfies criteria $K_1, K_2, \ldots, K_n$ respectively.

Each alternative A_i from the starting set of alternatives $\{A_1, A_2, \ldots, A_m\}$ is compared with the hypothetical alternative *A*. Using appropriate calculation, its position with respect to this hypothetical alternative, whether it is better than the hypothetical alternative or not, is determined. Also, we calculate how much each alternative A_i is better or worse than the hypothetical alternative *A*, and in that way we decide the final rank of alternatives from the starting set of alternatives $\{A_1, A_2, \ldots, A_m\}$.

In a multiplicative multi-criteria value model, the decision-maker's preference relation to the set of alternatives is given by the function v_n which maps the set of alternatives $\{A_1, A_2, \ldots, A_m\}$ into real numbers. For each alternative A_i, we have

$$v_n(A_i) = \left(1 + \frac{a_{i1} - a_1}{a_1} \cdot \rho_1\right) \cdot \left(1 + \frac{a_{i2} - a_2}{a_2} \cdot \rho_2\right) \cdots \left(1 + \frac{a_{in} - a_n}{a_n} \cdot \rho_n\right). \qquad (1)$$

This can be expressed as

$$v_n(A_i) = \prod_{k=1}^{n} \left(1 + \frac{a_{ik} - a_k}{a_k} \cdot \rho_k\right). \qquad (2)$$

If A_p and A_q are two alternatives from the starting set of alternatives $\{A_1, A_2, \ldots, A_m\}$, then we say that alternative A_p is preferred over alternative A_q if and only if $v_n(A_p) > v_n(A_q)$. For this, we use the following notation:

$$A_p \succ A_q \iff v_n(A_p) > v_n(A_q). \qquad (3)$$

If $v_n(A_p) = v_n(A_q)$ for two alternatives A_p and A_q, then we can omit the last criterion (which is the criterion of the lowest importance for the decision since all criteria are arranged in descending order) and the function v_{n-1} is given by

Maejo Int. J. Sci. Technol. **2016**, *10*(02), 233-241

$$v_{n-1}(A_i) = \prod_{k=1}^{n-1}\left(1 + \frac{a_{ik} - a_k}{a_k}\cdot\rho_k\right), \tag{4}$$

for any A_i from the starting set of alternatives $\{A_1, A_2, \ldots, A_m\}$.

Therefore, in this case we say that A_p is preferred over alternative A_q if and only if $v_n(A_p) = v_n(A_q)$ and $v_{n-1}(A_p) > v_{n-1}(A_q)$, and we use the following notation:

$$A_p \succ A_q \iff v_n(A_p) = v_n(A_q),\ v_{n-1}(A_p) > v_{n-1}(A_q). \tag{5}$$

Further, if $v_n(A_p) = v_n(A_q)$ and $v_{n-1}(A_p) = v_{n-1}(A_q)$ for two alternatives A_p and A_q, then we observe the function v_{n-2} defined by

$$v_{n-2}(A_i) = \prod_{k=1}^{n-2}\left(1 + \frac{a_{ik} - a_k}{a_k}\cdot\rho_k\right), \tag{6}$$

for any A_i from the starting set of alternatives $\{A_1, A_2, \ldots, A_m\}$, and the preference of alternative A_p over alternative A_q is defined by

$$A_p \succ A_q \iff v_n(A_p) = v_n(A_q),\ v_{n-1}(A_p) = v_{n-1}(A_q),\ v_{n-2}(A_p) > v_{n-2}(A_q). \tag{7}$$

Clearly, this procedure can be repeated and it ends when the first (the most important) criterion is taken into account.

Theorem 1. *Let A_p and A_q be two alternatives from the starting set of alternatives $\{A_1, A_2, \ldots, A_m\}$. Then A_p and A_q are indifferent if and only if they are identically equal, i.e. if $a_{pk} = a_{qk}$ holds for all $k = 1, 2, \ldots, n$.*

Proof: If $a_{pk} = a_{qk}$ holds for all $k = 1, 2, \ldots, n$, then clearly we have $v_1(A_p) = v_1(A_q),\ v_2(A_p) = v_2(A_q),\ \ldots,\ v_n(A_p) = v_n(A_q)$; thus, A_p and A_q are indifferent alternatives.

Conversely, let us suppose that A_p and A_q are indifferent alternatives, i.e. $v_1(A_p) = v_1(A_q),\ v_2(A_p) = v_2(A_q),\ \ldots,\ v_n(A_p) = v_n(A_q)$. Then from $v_1(A_p) = v_1(A_q)$ we have $1 + \frac{a_{p1} - a_1}{a_1}\cdot\rho_1 = 1 + \frac{a_{q1} - a_1}{a_1}\cdot\rho_1$. So we obtain $a_{p1} = a_{q1}$. Further, from $v_2(A_p) = v_2(A_q)$ and $a_{p1} = a_{q1}$ we have $a_{p2} = a_{q2}$. Repeating this procedure n times we obtain $a_{p1} = a_{q1}$, $a_{p2} = a_{q2}$, ... , $a_{p,n-1} = a_{q,n-1}$. Hence $v_n(A_p) = v_n(A_q)$ implies $a_{pn} = a_{qn}$. Thus, A_p and A_q are identically equal.

Theorem 2. *Let A_p be an alternative from the starting set of alternatives $\{A_1, A_2, \ldots, A_m\}$. Then $v_n(A_p) = 0$ if and only if $\rho_k = 1$ and $a_{pk} = 0$ for some $k \in \{1, 2, \ldots, n\}$.*

Proof: Assume that $v_n(A_p) = 0$. Then there exists $k \in \{1, 2, \ldots, n\}$ such that

$$1 + \frac{a_{pk} - a_k}{a_k}\cdot\rho_k = 0.$$

Thus, we have $(1-\rho_k)a_k + \rho_k a_{pk} = 0$, and since $a_k > 0, a_{pk} \geq 0$ and $0 < \rho_k \leq 1$, we get $(1-\rho_k)a_k = 0$ and $\rho_k a_{pk} = 0$. This gives us $\rho_k = 1$ and $a_{pk} = 0$.

Conversely, if for some $k \in \{1, 2, \ldots, n\}$, $\rho_k = 1$ and $a_{pk} = 0$, then we have

$$1 + \frac{a_{pk} - a_k}{a_k}\cdot\rho_k = 1 - 1 = 0.$$

Maejo Int. J. Sci. Technol. **2016**, *10*(02), 233-241

So it follows that $v_n(A_p) = 0$.

Corollary 1. *For arbitrary alternative* A_p*, we have* $v_n(A_p) \geq 0$.

Proof: This follows immediately from the proof of Theorem 2.

Corollary 2. *If alternative* A_p *does not satisfy an absolutely important criterion, then* $v_n(A_p) = 0$.

Proof: Let K_j be an absolutely important criterion; then $\rho_j = 1$. If alternative A_p does not satisfy criterion K_j, then $a_{pj} = 0$, and by Theorem 2 we obtain $v_n(A_p) = 0$.

It can be noticed that many multi-criteria decision methods which are based on an additive aggregation function allow some kind of compensation between criteria. The low performance of an important criterion can be redeemed in the overall aggregation by the good performance of a few other less important criteria. In view of Theorem 2 and Corollary 2, the multiplicative multi-criteria decision method provides that an important piece of information must be preserved. If an alternative does not satisfy an absolutely important criterion, then its overall aggregation value is zero.

Theorem 3. *The rank of alternatives from the set* $\{A_1, A_2, \ldots, A_m\}$ *obtained by the multiplicative method with respect to the given hypothetical alternative* $A(a_1, a_2, \ldots, a_n)$ *remains the same in the case that the starting set of alternatives is expanded by new alternatives* $\{B_1, B_2, \ldots, B_s\}$.

Proof: Using the iterative procedure defined and described by formulas (1) - (7), we obtain a partial-order relation on the arbitrary set of alternatives. Let $\succ$ denote the partial-order relation on the set $\{A_1, A_2, \ldots, A_m\}$ induced by functions v_n, and let $\triangleright$ denote the partial-order relation on the set $\{A_1, A_2, \ldots, A_m, B_1, B_2, \ldots, B_s\}$ induced by functions u_n. If A_p and A_q are two alternatives from the starting set of alternatives $\{A_1, A_2, \ldots, A_m\}$, then we have $v_n(A_p) = u_n(A_p)$ and $v_n(A_q) = u_n(A_q)$. So we have $A_p \succ A_q$ if and only if $A_p \triangleright A_q$.

It is well known that most of the multi-criteria decision methods suffer from a structuring problem in the sense that it is possible to obtain a reverse rank of alternatives by the introduction of new alternative options. By Theorem 3, the multiplicative multi-criteria decision method preserves that rank so that there are no possibilities of favouring or manipulating alternative ranking by taking new alternatives into account.

Corollary 3. *No alternative can be favoured by adding new alternatives.*

Proof: By Theorem 3, adding new alternatives to the multi-criteria model does not rearrange the rank of previously introduced alternative choices, so no alternative can be favoured.

APPLICATION - ADVERTISING OF GORNJA TREPCA SPA

In this section we give an application of the introduced multiplicative method. We picked the advertising of Gornja Trepca Spa and the choice of a celebrity person who would best suit the task.

The atomic Spa in Gornja Trepca, Serbia, is known outside of the borders of Serbia for the treatment of rheumatic and neurological diseases, thanks to the unique composition of its water, with an optimal content of rare elements such as cesium, lithium, strontium, cobalt and uranium. It is a type of mineral water with mild radioactive properties, which positively affects the human body. One of the advantages of this Spa, in addition to the healing water, is the expertise of the medical staff and the application of modern methods of treatment. To increase the number of guests who come for diagnosis and therapy, or as tourists who prefer a quiet place and walk, the management of the Spa considered promotion through TV, billboards, Internet, etc. It was noted that a celebrity person could provide an additional beneficial brand for the promoting campaign. In the selection of brand name, the following alternative choices were considered.

Alternatives

A_1 : A former athlete, a football player from one of the most famous clubs, who had several times played in the national team. Because of the meniscus surgery, he moves with difficulty and uses the Spa to relieve rheumatism.

A_2 : A retired actress, known for her roles in numerous films and TV series, who often played roles in dramas and performed as a comedian. Now she occasionally performs and comes to Spa for examination and treatment.

A_3 : A former athlete, a basketball player who played in several clubs and the national team. He comes to Spa because of the degenerative changes in the spine.

A_4 : A retired journalist who still occasionally gives reports on the local television. He uses the Spa because of the digestive problems and rheumatism.

Criteria

C_1 : The level of trust for the person who promotes the Spa.

C_2 : The level of positive visual impact of the person who promotes the Spa.

C_3 : The level of positive emotions elicited by the person who promotes the Spa.

C_4 : The level of credibility of the person who promotes the Spa.

The managers of the Spa regularly questioned the guests for information on their satisfaction of the Spa treatments. During May-September 2013, the interview list contained additional questions concerning advertising of the Spa; more than 1000 guests were questioned. According to the database provided by the representative of the Spa owners and doctors (two of them are managers of the Spa), we had obtained a set of values by statistical analysis of the database, i.e. the values of alternatives A_1, A_2, A_3, A_4 with respect to criteria C_1, C_2, C_3, C_4 presented by a decision matrix given in Table 2. The values of degree of criteria importance (Table 3) and those of the reference point representing the acceptable suitable values of this case study (Table 4) were also obtained from results of this questioning and from the suggestions of the Spa team.

Table 2. Decision matrix of the case study

	K_1	K_2	K_3	K_4
A_1	0.80	0.60	0.80	0.75
A_2	0.90	0.85	0.95	0.70
A_3	0.90	0.55	0.90	0.80
A_4	0.80	0.90	0.60	0.60

Table 3. Degree of importance of each criterion

K_1	K_2	K_3	K_4
1.00	0.90	0.80	0.70

Table 4. Reference points

	K_1	K_2	K_3	K_4
A	0.80	0.70	0.80	0.70

Using formula (2), we obtained the values of function v_4 for the set of alternatives A_1, A_2, A_3, A_4. These values are given in Table 5.

Table 5. Values of v_4 for the alternatives

	v_4
A_1	0.915
A_2	1.543
A_3	1.096
A_4	0.905

Thus, by (3) we have $A_2 \succ A_3 \succ A_1 \succ A_4$, meaning that A_2 is the best alternative choice. Since the difference $v_4(A_1) - v_4(A) = 0.543$, we can conclude that A_2 significantly overlaps the acceptable suitable values. The differences between A_1, A_3, A_4 and A are -0.085, 0.096 and -0.095 respectively, so all these alternatives are close to the reference points and are a good choice of celebrity person for Spa advertising but alternative A_2 is much better. Thus, our suggestion to the Spa management is the alternative choice A_2.

CONCLUSIONS

In this paper we have introduced and studied the multiplicative multi-criteria decision method. The overall value of aggregation function calculated by this method preserves information on the performance of highly important criteria and adding new alternatives does not rearrange previously introduced alternatives. The parameters of the aggregation function (weights and utility functions) will be the subject of study in our further research.

ACKNOWLEDGEMENTS

The authors wish to thank the reviewers and editors for helpful comments and suggestions that improve the quality of the article.

REFERENCES

1. C. L. Hwang and K. Yoon, "Multiple Attribute Decision Making Methods and Applications", Spriner-Verlag, Berlin, **1981**.
2. G. Choquet, "Theory of capacities", *Annal. l'Inst. Fourier*, **1954**, *5*, 131-295.
3. M. Zizovic, N. Damljanovic, V. Lazarevic and N. Deretic, "New method for multicriteria analysis", *U.P.B. Sci. Bull. A*, **2011**, *73*, 13-22.
4. M. Radojicic, M. Zizovic, Z. Nesic and J. V. Vasovic, "Modified approach to PROMETHEE for multi-criteria decision-making", *Maejo Int. J. Sci. Technol.*, **2013**, *7*, 408-421.
5. W. K. Brauers, "Optimization Methods for a Stakeholder Society—a Revolution in Economic Thinking by Multi-objective Optimization: Nonconvex Optimization and Its Applications", Kluwer Academic Publishers, Boston, **2004**.
6. J. P. Brans and Ph. Vincke, "A preference ranking organization method", *Manage. Sci.*, **1985**, *31*, 647-656.

7. D. Dubois and H. Prade, "Possibility theory as a basis for qualitative decision theory", Proceedings of 14th International Joint Conference on Artifcial Intelligence, **1995**, Montreal, Canada, pp.1924-1930.
8. J. Figueira, Y. de Smet and J. P. Brans, "MCDA methods for sorting and clustering problems: Promethee TRI and Promethee CLUSTER", Technical Report TR/SMG/2004/02, Universite libre de Bruxelles, Belgium, **2004**.
9. B. Ivanovic, "A method of establishing a list of development indicators", Study XXIV, UNESCO, Paris, **1973**.
10. G. Munda, "Multicriteria Evaluation in a Fuzzy Environment", Physica-Verlag, Heidleberg, **1995**.
11. B. Roy, "Multicriteria Methodology for Decision Aiding", Kluwer Academic Publishers, Dordecht, **1996**.
12. W. Edwards, "How to use multiattribute utility measurement for social decisionmaking", *IEEE Trans. Syst. Man Cybern.*, **1977**, *7*, 326-340.
13. F. X. Frei, R. Kalakota, A. J. Leone and L. M. Marx, "Process variation as a determinant of the bank performance: Evidence from the retail banking study", *Manage. Sci.*, **1999**, *45*, 1210-1220.
14. F. Garcia, F. Guijarro and I. Moya, "A goal programming approach to estimating performance weights for ranking firms", *Comput. Operat. Res.*, **2010**, *37*, 1597-1609.
15. A. Jahan, F. Mustapha, S. M. Sapuan, M. Y. Ismail and M. Bahraminasab, "A framework for weighting of criteria in ranking stage of material selection process", *Int. J. Adv. Manuf. Technol.*, **2012**, *58*, 411-420.
16. I. B. Huang, J. Keisler and I. Linkov, "Multi-criteria decision analysis in environmental sciences: Ten years of applications and trends", *Sci. Total Environ.*, **2011**, *409*, 3578-3594.
17. C. Macharis, A. Verbeke, K. Brucker, "The strategic evaluation of new technologies through multi-criteria analysis: The advisors case", in "Economic Impacts of Intelligent Transportation Systems: Innovations and case studies" (Ed. E. Bekiaris and Y. J. Nakanishi), Elsevier, Amsterdam, **2004**.
18. S. Opricovic, "Multicriteria optimization in civil engineering", Monograph, Faculty of Civil Engineering, University of Belgrade, Belgrade, Serbia, **1998** (in Serbian).
19. S. Opricovic, "A compromise solution in water resources planning", *Water Resour. Manage.*, **2009**, *23*, 1549-1561.
20. H. Polatidis, D. A. Haralambopoulos, G. Munda and R.Vreeker, "Selecting an appropriate multi-criteria decision analysis technique for renewable energy planning", *Ener. Sour. B*, **2006**, *1*, 181-193.
21. T. L. Saaty and L. G. Vargas, "Decision Making with the Analytic Network Process: Economic, Political, Social and Technological Applications with Benefits, Opportunities, Costs and Risks", Springer, New York, **2006**.
22. M. T. Tabucanon, "Multiple Criteria Decision Making in Industry", Elsevier, Amsterdam, **1988**.
23. O. S. Vaidya and S. Kumar, "Analytic hierarchy process: An overview of applications", *European J. Operat. Res.*, **2006**, *169*, 1-29.
24. D. von Winterfeldt and W. Edwards, "Decision Analysis and Behavioral Research", Cambrigde University Press, Cambridge, **1986**.

Maejo Int. J. Sci. Technol. **2016**, *10*(02), 233-241

25. T. L. Saaty, "Decision Making for Leaders: The Analytical Hierarchy Process for Decisions in a Complex World", Wadsworth Publishing, Belmont (CA), **1982**.
26. J. Figueira, S. Greco and M. Ehrgott, "Multiple Criteria Decision Analysis: State of the Art Surveys", Springer-Verlag, New York, **2005**.
27. M. Radojicic and M. R. Zizovic, "Applications of methods of multi-criteria analysis in business decision-making", Monograph, Technical Faculty in Cacak, University of Kragujevac, Serbia, **1998** (in Serbian).
28. E. Triantaphyllou, "Multi-Criteria Decision Making Methods: A Comparative Study", Kluwer Academic Publishers, Dordrecht, **2000**.
29. M. Zeleny, "Multiple Criteria Decision Making", McGrawHill, New York, **1982**.
30. M. Zizovic and N. Damljanovic, "Main advantage of lattice MCD-method", Proceedings of 13th International Conference on Mathematics and Its Applications, **2012,** Timisoara, Romania, pp.315-320.