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Full Paper

Similarity measures between temporal intuitionistic fuzzy sets

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Abstract: A temporal intuitionistic fuzzy set (TIFS), which is an extended version of an intuitionistic fuzzy set, is more applicable for representing spatio-temporal aspects. One of the most prominent concepts that we need in its applications is the similarity measure between TIFSs. In the present study we propose cosine similarity measure between two TIFSs. A comparative example shows that cosine similarity is reasonable and attains satisfactory performance on spatio-temporal pattern recognition problems and medical diagnosis.

Keywords: intuitionistic fuzzy set, temporal intuitionistic fuzzy set, cosine similarity measure

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INTRODUCTION

The theory of fuzzy sets [1] is one of the most important concepts in the set theory. It has been well understood and used in various aspects of science and technology such as medicine, engineering and computer sciences. As a natural generalisation of fuzzy sets, Atanassov [2] proposed the concept of intuitionistic fuzzy sets where each element has two degrees named as membership and non-membership degrees. The application of intuitionistic fuzzy sets in place of fuzzy sets means another degree of freedom introduced into the set description. Such fuzzy set generalisation gives us an extra possibility of representing imperfect knowledge, which leads to describing many problems in a more convenient way.

Having their applications in various fields, similarity measures quantify the extent to which two different sets are alike. Researchers are interested in checking the similarity degree between two patterns or images: Are they identical or approximately identical? Or at least to what extent are

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they identical? The two comparable sets may be fuzzy sets, intuitionistic fuzzy sets, vague sets, etc. Many similarity measures between intuitionistic fuzzy sets have been proposed in the literature [3-11].

In the real world many situations like weather, medicine, economy and image-video processing are spatio-temporal. In 1991 Atanassov [12] initiated the concept of temporal intuitionistic fuzzy set (TIFS), in which the membership and non-membership degrees of an element vary with both the element and the time moment. By using the new TIFS theory many spatio-temporal situations in the real world can be handled in a more realistic and effective manner. Usually decisions are made based on the amount of information available at a specific time, so decision time is a crucial factor in its quality, especially in uncertain environments. Chen and Tu [13] proposed time-validated intuitionistic fuzzy sets based on TIFSs to make an earlier decision based on the desired information level. Moreover, they used some numerical examples to confirm the applicability of the proposed theorems and measures. The TIFS theory is an almost untouched area and most of its similarity measures have not been proposed yet.

In 2016 Kultu et al. [14] proposed the temporal intuitionistic fuzzy distance, overall intuitionistic fuzzy distance, temporal intuitionistic fuzzy similarity measure, temporal intuitionistic fuzzy entropy and temporal intuitionistic fuzzy inclusion measure. Moreover, the major properties and relationships between these measures were studied and investigated. As a continuation of this work, we propose and extend the cosine similarity measure to the TIFS theory. Furthermore, we compare the proposed cosine similarity with the already defined similarity measures between TIFSs. Finally, we apply the cosine similarity to pattern recognition and medical diagnosis.

TIFSs

This section is devoted to briefly reviewing the concept and notion of the TIFS theory. Moreover, we extend the theory with some concepts that are essential for establishing the main issues of the paper.

Let X be a universe, T be a non-empty set of `time moments' and $A \subset X$. A TIFS is defined by $A = \{((x,t), \mu_A(x,t), \nu_A(x,t)) : (x,t) \in A \times T\}$, where $\mu_A : A \times T \to [0,1]$ and $\nu_A : A \times T \to [0,1]$ such that

$$0 \le \mu_A(x,t) + \nu_A(x,t) \le 1,$$

$\mu_A(x,t)$ and $\nu_A(x,t)$ being the degrees of membership and non-membership respectively of the element $x \in A$ at the time moment $t \in T$. The hesitation degree of a TIFS, A, is defined by

$$\pi_A(x,t) = 1 - \mu_A(x,t) - \nu_A(x,t).$$

It is obvious that $0 \le \pi(x,t) \le 1$ for each $(x,t) \in A \times T$. For more information about the TIFSs, we refer to Mitchell [6].

Let A and B be two TIFSs defined on the universe of discourse $X = \{x_1, x_2, \dots, x_n\}$ and time moments $T = \{t_1, t_2, \dots, t_m\}$. The correlation coefficient of A and B is given by:

$$\kappa(A,B) = \frac{C(A,B)}{\sqrt{T(A).T(B)}},$$

where

$$C(A,B) = \sum_{i=1}^{n} \sum_{j=1}^{m} \left(\mu_A(x_i,t_j)\mu_B(x_i,t_j) + \nu_A(x_i,t_j)\nu_B(x_i,t_j) \right)$$

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is the correlation of the two TIFSs A and B, and

$$T(A) = \sum_{i=1}^{n} \sum_{j=1}^{m} \left(\mu_A^2(x_i, t_j) + \nu_A^2(x_i, t_j)\right), \quad T(B) = \sum_{i=1}^{n} \sum_{j=1}^{m} \left(\mu_B^2(x_i, t_j) + \nu_B^2(x_i, t_j)\right)$$

are the informational temporal intuitionistic energies of A and B respectively. Obviously, $T(A) = T(A')$. Moreover, the correlation of the two TIFSs A and B satisfies the following properties:

(1) $C(A, A) = T(A)$;

(2) $C(A, B) = C(B, A)$.

Theorem 1. *Let A and B be two TIFSs in the universe of discourse X and time moments T. Then*

(1) $\kappa(A, B) = 1$ if $A = B$;

(2) $\kappa(A, B) = \kappa(B, A)$;

(3) $0 \leq \kappa(A, B) \leq 1$.

Proof. It is obvious that κ satisfies (1) and (2). For (3), the inequality $\kappa(A, B) \geq 0$ is evident. We will prove that $\kappa(A, B) \leq 1$. Suppose that

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \mu_A^2(x_i, t_j) = \alpha_1, \quad \sum_{i=1}^{n} \sum_{j=1}^{m} \mu_B^2(x_i, t_j) = \alpha_2,$$

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \nu_A^2(x_i, t_j) = \alpha_3, \quad and \quad \sum_{i=1}^{n} \sum_{j=1}^{m} \nu_B^2(x_i, t_j) = \alpha_4.$$

Then

$$\kappa(A, B) = \frac{C(A, B)}{\sqrt{T(A).T(B)}}$$

$$= \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} \left(\mu_A(x_i, t_j)\mu_B(x_i, t_j) + \nu_A(x_i, t_j)\nu_B(x_i, t_j)\right)}{\left[\sum_{i=1}^{n} \sum_{j=1}^{m} \left(\mu_A^2(x_i, t_j) + \nu_A^2(x_i, t_j)\right). \sum_{i=1}^{n} \sum_{j=1}^{m} \left(\mu_B^2(x_i, t_j) + \nu_B^2(x_i, t_j)\right)\right]^{\frac{1}{2}}}$$

$$\leq \frac{\sum_{i=1}^{n} \sum_{j=1}^{m} \mu_A(x_i, t_j)\mu_B(x_i, t_j) + \sum_{i=1}^{n} \sum_{j=1}^{m} \nu_A(x_i, t_j)\nu_B(x_i, t_j)}{\left(\sum_{i=1}^{n} \sum_{j=1}^{m} \mu_A^2(x_i, t_j) + \sum_{i=1}^{n} \sum_{j=1}^{m} \nu_A^2(x_i, t_j)\right)^{\frac{1}{2}}. \left(\sum_{i=1}^{n} \sum_{j=1}^{m} \mu_B^2(x_i, t_j) + \sum_{i=1}^{n} \sum_{j=1}^{m} \nu_B^2(x_i, t_j)\right)^{\frac{1}{2}}}$$

$$= \frac{(\alpha_1.\alpha_2)^{\frac{1}{2}} + (\alpha_3.\alpha_4)^{\frac{1}{2}}}{\left[(\alpha_1 + \alpha_3)^{\frac{1}{2}}.(\alpha_2 + \alpha_4)^{\frac{1}{2}}\right]},$$

i.e.

$$\kappa^2(A, B) \leq \frac{\alpha_1\alpha_2 + 2(\alpha_1\alpha_2.\alpha_3\alpha_4)^{\frac{1}{2}} + \alpha_3\alpha_4}{(\alpha_1 + \alpha_3).(\alpha_2 + \alpha_4)}.$$

But

$$\kappa^2(A, B) - 1 \leq \frac{\alpha_1\alpha_2 + 2(\alpha_1\alpha_2.\alpha_3\alpha_4)^{\frac{1}{2}} + \alpha_3\alpha_4}{(\alpha_1 + \alpha_3).(\alpha_2 + \alpha_4)} - 1$$

$$= \frac{\alpha_1\alpha_2 + 2(\alpha_1\alpha_2.\alpha_3\alpha_4)^{\frac{1}{2}} + \alpha_3\alpha_4 - (\alpha_1 + \alpha_3).(\alpha_2 + \alpha_4)}{(\alpha_1 + \alpha_3).(\alpha_2 + \alpha_4)}$$

$$= -\frac{\left[(\alpha_1\alpha_4)^{\frac{1}{2}} - (\alpha_2\alpha_3)^{\frac{1}{2}}\right]^2}{(\alpha_1 + \alpha_3).(\alpha_2 + \alpha_4)}$$

$$\leq 0.$$

Hence $\kappa^2(A,B) \leq 1$; that is, $\kappa(A,B) \leq 1$.

Definition 1. *Let* $S: TIFSs(X,T) \times TIFSs(X,T) \to [0,1]$ *be a function where* $TIFSs(X,T)$ *is the family of TIFSs in the universe discourse* X *and time moments set* T. *Then* $S(A,B)$ *is said to be the similarity degree between* $A, B \in TIFSs(X,T)$ *if* $S(A,B)$ *satisfies the following statements:*

(1) $0 \leq S(A,B) \leq 1$;
(2) $S(A,B) = 1$ if $A = B$;
(3) $S(A,B) = S(B,A)$;
(4) $S(A,C) \leq S(A,B)$ and $S(A,C) \leq S(B,C)$ if $A \subseteq B \subseteq C$ and $C \in TIFSs(X,T)$.

A CONSTRUCTIVE METHOD FOR TIFSs

The TIFS theory is the only suitable tool for dealing with the imperfect spatio-temporal information. The construction of membership and non-membership functions of TIFS is a difficult task and poses challenges to reseachers. In this section we extend the method proposed by Chaira [15] for intuitionistic fuzzy sets based on the Sugeno intuitionistic fuzzy generator [16]. If $\mu(x,t)$ is the membership function of the temporal fuzzy set A, then the non-membership function $\nu_A(x,t) = G(\mu_A(x,t))$, where

$$G(\mu_A(x,t)) = \frac{1-\ \mu_A(x,t)}{1+\alpha\ \mu_A(x,t)}, \qquad \lambda > 0,$$

and $G(1) = 0$, $G(0) = 1$. With the help of the Sugeno intuitionistic fuzzy generator, the TIFS A is given by

$$A^\alpha = \left\{\langle (x,t),\ \mu_A(x,t), \frac{1-\ \mu_A(x,t)}{1+\alpha\ \mu_A(x,t)}\rangle \mid (x,t) \in\ X \times T\right\},$$

and the hesitation degree is

$$\pi_{A^\alpha}(x,t) = 1 - \mu_A(x,t) - \frac{1-\mu_A(x,t)}{1+\alpha\ \mu_A(x,t)}.$$

It was observed that with an increase in α, the fuzzy complement or the Sugeno generator decreases. Thus, the increase in the non-membership value enhances the hesitation degree.

Example. Suppose that A is a temporal fuzzy set (Table 1) defined on $X = \{x_1, x_2, x_3\}$ with respect to the time moment set $T = \{t_1, t_2\}$:

Table 1. Temporal fuzzy set A

x \ t	t_1	t_2
x_1	1.0	0.7
x_2	0.8	0.5
x_3	0.7	0.0

If $\alpha = 1$, then by using the Sugeno intuitionistic fuzzy generator, the TIFS A^1 is given by (Table 2):

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Table 2. TIFS A^1

t \ x	t_1	t_2
x_1	$< 1.0,0 >$	$< 0.7,0.18 >$
x_2	$< 0.8,0.11 >$	$< 0.5,0.33 >$
x_3	$< 0.7,0.18 >$	$< 0.0,1.0 >$

and the hesitation degrees are (Table 3):

Table 3. Hesitation degrees

t \ x	t_1	t_2
x_1	0.0	0.12
x_2	0.09	0.17
x_3	0.12	0.0

COSINE SIMILARITY MEASURE FOR TIFSs

The similarity measure as proposed by us is presented in this section. Suppose there are two TIFSs A and B defined on $X = \{x_1, x_2, \dots, x_n\}$ and the time moment set $T = \{t_1, t_2, \dots, t_m\}$; a cosine similarity measure between A and B is proposed as follows:

$$C_T(A,B) = \frac{1}{mn}\sum_{i=1}^{n}\sum_{j=1}^{m}\frac{\mu_A(x_i,t_j)\mu_B(x_i,t_j)+\nu_A(x_i,t_j)\nu_B(x_i,t_j)}{\sqrt{\mu_A^2(x_i,t_j)+\nu_A^2(x_i,t_j)}.\sqrt{\mu_B^2(x_i,t_j)+\nu_B^2(x_i,t_j)}}. \tag{1}$$

In the case where $n = m = 1$, the cosine similarity measure between A and B is the same as the correlation coefficient between TIFSs A and B, i.e. $C_T(A,B) = k(A,B)$. The following properties hold good for C_T:

(1) $0 \le C_T(A,B) \le 1$;
(2) $C_T(A,B) = C_T(B,A)$;
(3) $C_T(A,B) = 1$ if $A = B$.

Proof. It is obvious that C_T satisfies (1) and (2). If $A = B$, then $\mu_A(x_i,t_j) = \mu_B(x_i,t_j)$ and $\nu_A(x_i,t_j) = \nu_B(x_i,t_j)$ for each $i = 1,2,\dots,n$ and $j = 1,2,\dots,m$. Hence $C_T(A,B) = 1$.

For two TIFSs A and B defined on a universe X and time moments set T, we define the distance measure of the angle as

$$d(A,B) = \arccos(C_T(A,B)).$$

The following properties hold:

(1) If $0 \le C_T(A,B) \le 1$, then $d(A,B) \ge 0$;
(2) If $C_T(A,B) = 1$, then $d(A,B) = 0$;
(3) If $C_T(A,B) = C_T(B,A)$, then $d(A,B) = d(B,A)$;
(4) If $A \subseteq B \subseteq C$, then $d(A,C) \le d(A,B) + d(B,C)$.

Proof. It is obvious that $d(A,B)$ satisfies (1), (2) and (3). For (4), let us consider the distance measures of the angle between the vectors $A(x_i,t_j)$, $B(x_i,t_j)$ and $C(x_i,t_j)$:

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$$d_{(i,j)}(A(x_i,t_j),B(x_i,t_j)) = \arccos\left(C_T\left(A(x_i,t_j),B(x_i,t_j)\right)\right),$$
$$d_{(i,j)}(B(x_i,t_j),C(x_i,t_j)) = \arccos\left(C_T\left(B(x_i,t_j),C(x_i,t_j)\right)\right),$$
$$d_{(i,j)}(A(x_i,t_j),C(x_i,t_j)) = \arccos\left(C_T\left(A(x_i,t_j),C(x_i,t_j)\right)\right),$$

where $i = 1,2,\dots,n$, $j = 1,2,\dots,m$ and

$$C_T\left(A(x_i,t_j),B(x_i,t_j)\right) = \frac{\mu_A(x_i,t_j)\mu_B(x_i,t_j) + \nu_A(x_i,t_j)\nu_B(x_i,t_j)}{\sqrt{\mu_A^2(x_i,t_j)+\nu_A^2(x_i,t_j)}.\sqrt{\mu_B^2(x_i,t_j)+\nu_B^2(x_i,t_j)}},$$
$$C_T\left(B(x_i,t_j),C(x_i,t_j)\right) = \frac{\mu_B(x_i,t_j)\mu_C(x_i,t_j) + \nu_B(x_i,t_j)\nu_C(x_i,t_j)}{\sqrt{\mu_B^2(x_i,t_j)+\nu_B^2(x_i,t_j)}.\sqrt{\mu_C^2(x_i,t_j)+\nu_C^2(x_i,t_j)}},$$
$$C_T\left(A(x_i,t_j),C(x_i,t_j)\right) = \frac{\mu_A(x_i,t_j)\mu_C(x_i,t_j) + \nu_A(x_i,t_j)\nu_C(x_i,t_j)}{\sqrt{\mu_A^2(x_i,t_j)+\nu_A^2(x_i,t_j)}.\sqrt{\mu_C^2(x_i,t_j)+\nu_C^2(x_i,t_j)}}.$$

For the three vectors $A(x_i,t_j)$, $B(x_i,t_j)$ and $C(x_i,t_j)$, if $A(x_i,t_j) \subseteq B(x_i,t_j) \subseteq C(x_i,t_j)$ for each $i = 1,2,\dots,n$ and $j = 1,2,\dots,m$, then

$$d_{(i,j)}(A(x_i,t_j),C(x_i,t_j)) \le d_{(i,j)}(A(x_i,t_j),B(x_i,t_j)) + d_{(i,j)}(B(x_i,t_j),C(x_i,t_j)).$$

Substituting in the definition of C_T, we can get $d(A,C) \le d(A,B) + d(B,C)$. This completes the proof.

If $w(i,j)$ is the weight of (x_i,t_j), where $w(i,j) \in [0,1]$, $i = 1,2,\dots,n$, $j = 1,2,\dots,m$ and $\sum_{i=1}^{n}\sum_{j=1}^{m} w(i,j) = 1$, the weighted cosine similarity measure between TIFSs A and B is proposed as follows:

$$W_T(A,B) = \sum_{i=1}^{n}\sum_{j=1}^{m} w(i,j)\frac{\mu_A(x_i,t_j)\mu_B(x_i,t_j)+\nu_A(x_i,t_j)\nu_B(x_i,t_j)}{\sqrt{\mu_A^2(x_i,t_j)+\nu_A^2(x_i,t_j)}.\sqrt{\mu_B^2(x_i,t_j)+\nu_B^2(x_i,t_j)}}. \tag{2}$$

In the case where $\sum_{i=1}^{n}\sum_{j=1}^{m} w(i,j) = \frac{1}{nm}$, the expression (2) becomes the cosine similarity measure between A and B. It is worth noticing that $W_T(A,B)$ satisfies the following properties:

(1) $0 \le W_T(A,B) \le 1$;

(2) $W_T(A,B) = W_T(B,A)$;

(3) $W_T(A,B) = 1$ if $A = B$.

COMPARATIVE EXAMPLES

In this section we compare the proposed cosine similarity measure and other similarity measures in the TIFS. Since the TIFS theory has not yet been explored fully, most of the similarity measures have not been defined. We therefore propose the following similarity measures between two TIFSs to make our comparison. Let A and B be two TIFSs in the universe discourse $X = \{x_1, x_2, \dots, x_n\}$ and time moment set $T = \{t_1, t_2, \dots, t_m\}$. Then we have the following degrees of similarity between A and B:

$$S_C(A,B) = 1 - \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}|S_A(i,j) - S_B(i,j)|}{2mn}, \tag{3}$$

where $S_A(i,j) = \mu_A(x_i,t_j) - \nu_A(x_i,t_j)$ and $S_B(i,j) = \mu_B(x_i,t_j) - \nu_B(x_i,t_j)$;

$$S_H(A,B) = 1 - \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\left(|\mu_A(x_i,t_j)-\mu_B(x_i,t_j)|+|\nu_A(x_i,t_j)-\nu_B(x_i,t_j)|\right)}{2mn}; \tag{4}$$

$$S_0(A,B) = 1 - \sqrt{\frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\mu_A(x_i,t_j)-\mu_B(x_i,t_j)\right)^2+\left(\nu_A(x_i,t_j)-\nu_B(x_i,t_j)\right)^2}{2mn}}; \tag{5}$$

$$S_D(A,B) = 1 - \frac{1}{\sqrt[p]{mn}}\sqrt[p]{\sum_{i=1}^{n}\sum_{j=1}^{m}|\phi_A(i,j)-\phi_B(i,j)|^p}, \tag{6}$$

where $\phi_A(i,j) = \frac{\mu_A(x_i,t_j)+1-\nu_A(x_i,t_j)}{2}$, $\phi_B(i,j) = \frac{\mu_B(x_i,t_j)+1-\nu_B(x_i,t_j)}{2}$, and $1 \le p < +\infty$;

$$S_r(A,B) = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\min\{\mu_A(x_i,t_j),\mu_B(x_i,t_j)\}+\min\{1-\nu_A(x_i,t_j),1-\nu_B(x_i,t_j)\}\right)}{\sum_{i=1}^{n}\sum_{j=1}^{m}\left(\max\{\mu_A(x_i,t_j),\mu_B(x_i,t_j)\}+\max\{1-\nu_A(x_i,t_j),1-\nu_B(x_i,t_j)\}\right)}. \tag{7}$$

It is noteworthy that the above measures (3), (4), (5), (6) and (7) satisfy the four statements given in Definition 1. To demonstrate the reasonability of the proposed similarity measures, we consider the following two TIFSs (Tables 4 and5):

Table 4. TIFS *A*

x \ t	t_1	t_2	t_3	t_4	t_5	t_6
x_1	$\langle 0.3,0.3\rangle$	$\langle 0.3,0.4\rangle$	$\langle 0.4,0.5\rangle$	$\langle 0.5,0.5\rangle$	$\langle 0.5,0.4\rangle$	$\langle 0.3,0.2\rangle$
x_2	$\langle 0.3,0.4\rangle$	$\langle 1,0\rangle$	$\langle 0.4,0.2\rangle$	$\langle 0.6,0.1\rangle$	$\langle 0,0.7\rangle$	$\langle 0.2,0.2\rangle$
x_3	$\langle 1,0\rangle$	$\langle 0.7,0.3\rangle$	$\langle 0.5,0.1\rangle$	$\langle 0.1,0.6\rangle$	$\langle 0.3,0.3\rangle$	$\langle 0.5,0.2\rangle$

Table 5. TIFS *B*

x \ t	t_1	t_2	t_3	t_4	t_5	t_6
x_1	$\langle 0.2,0.4\rangle$	$\langle 0.2,0.1\rangle$	$\langle 0.3,0.5\rangle$	$\langle 0.3,0.6\rangle$	$\langle 0.6,0.2\rangle$	$\langle 0.1,0.5\rangle$
x_2	$\langle 0.3,0.3\rangle$	$\langle 0.5,0.5\rangle$	$\langle 0.4,0.2\rangle$	$\langle 0.2,0.6\rangle$	$\langle 0.1,0.7\rangle$	$\langle 0.3,0.3\rangle$
x_3	$\langle 1,0\rangle$	$\langle 0.8,0.1\rangle$	$\langle 0.9,0.1\rangle$	$\langle 0.2,0.6\rangle$	$\langle 0.4,0.3\rangle$	$\langle 0.2,0.2\rangle$

A comparison between the results from formulas (1), (3), (4), (5), (6) and (7) are illustrated numerically in the TIFS theory as shown in Table 6, which shows that the cosine similarity measure C_T is much better in comparison to others.

Table 6. Comparison between cosine similarity measure and other similarity measures

x \ t	t_1	t_2	t_3	t_4	t_5	t_6
x_1	$S_C = 0.9000$	$S_C = 0.9000$	$S_C = 0.9500$	$S_C = 0.8500$	$S_C = 0.8500$	$S_C = 0.7500$
	$S_H = 0.9000$	$S_H = 0.8000$	$S_H = 0.9500$	$S_H = 0.8500$	$S_H = 0.8500$	$S_H = 0.7500$
	$S_0 = 0.9000$	$S_0 = 0.7764$	$S_0 = 0.9293$	$S_0 = 0.8419$	$S_0 = 0.8419$	$S_0 = 0.7450$

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Table 6 (continued).

t \ x	t_1	t_2	t_3	t_4	t_5	t_6
	$S_D = 0.9000$	$S_D = 0.9000$	$S_D = 0.9500$	$S_D = 0.8500$	$S_D = 0.8500$	$S_D = 0.7500$
	$S_r = 0.8000$	$S_r = 0.6667$	$S_r = 0.8889$	$S_r = 0.7000$	$S_r = 0.7857$	$S_r = 0.5455$
	$C_T = 0.9487$	$C_T = 0.8944$	$C_T = 0.9910$	$C_T = 0.9487$	$C_T = 0.9383$	$C_T = 0.7071$
x_2	$S_C = 0.9500$	$S_C = 0.5000$	$S_C = 1.0000$	$S_C = 0.5500$	$S_C = 0.9500$	$S_C = 1.0000$
	$S_H = 0.9500$	$S_H = 0.5000$	$S_H = 1.0000$	$S_H = 0.5500$	$S_H = 0.9500$	$S_H = 0.9000$
	$S_0 = 0.9293$	$S_0 = 0.5000$	$S_0 = 1.0000$	$S_0 = 0.5472$	$S_0 = 0.9293$	$S_0 = 0.9000$
	$S_D = 0.9500$	$S_D = 0.5000$	$S_D = 1.0000$	$S_D = 0.5500$	$S_D = 0.9500$	$S_D = 1.0000$
	$S_r = 0.9000$	$S_r = 0.5000$	$S_r = 1.0000$	$S_r = 0.4000$	$S_r = 0.7500$	$S_r = 0.8182$
	$C_T = 0.9899$	$C_T = 0.7071$	$C_T = 1.0000$	$C_T = 0.4679$	$C_T = 0.9899$	$C_T = 1.0000$
x_3	$S_C = 1.0000$	$S_C = 0.8500$	$S_C = 0.8000$	$S_C = 0.9500$	$S_C = 0.9500$	$S_C = 0.8500$
	$S_H = 1.0000$	$S_H = 0.8500$	$S_H = 0.8000$	$S_H = 0.9500$	$S_H = 0.9500$	$S_H = 0.8500$
	$S_0 = 1.0000$	$S_0 = 0.8419$	$S_0 = 0.7172$	$S_0 = 0.9293$	$S_0 = 0.9293$	$S_0 = 0.7879$
	$S_D = 1.0000$	$S_D = 0.8500$	$S_D = 0.8000$	$S_D = 0.9500$	$S_D = 0.9500$	$S_D = 0.8500$
	$S_r = 1.0000$	$S_r = 0.8235$	$S_r = 0.7778$	$S_r = 0.8333$	$S_r = 0.9091$	$S_r = 0.7692$
	$C_T = 1.0000$	$C_T = 0.9609$	$C_T = 0.9962$	$C_T = 0.9878$	$C_T = 0.9899$	$C_T = 0.9191$

APPLICATION IN PATTERN RECOGNITION AND MEDICAL DIAGNOSIS

Let us assume that there exist three well known patterns P_1, P_2 and P_3, which are represented by the following three TIFSs (Tables 7-9) in the given discourse $X = \{x_1, x_2, x_3\}$ and time moments set $T = \{t_1, t_2\}$:

Table 7. TIFS P_1

t \ x	t_1	t_2
x_1	$\langle 1.0,0.0 \rangle$	$\langle 0.7,0.2 \rangle$
x_2	$\langle 0.8,0.0 \rangle$	$\langle 0.5,0.5 \rangle$
x_3	$\langle 0.7,0.1 \rangle$	$\langle 0.0,0.8 \rangle$

Table 8. TIFS P_2

t \ x	t_1	t_2
x_1	$\langle 0.8,0.1 \rangle$	$\langle 0.4,0.3 \rangle$
x_2	$\langle 1.0,0.0 \rangle$	$\langle 0.9,0.0 \rangle$
x_3	$\langle 0.9,0.0 \rangle$	$\langle 0.7,0.1 \rangle$

Table 9. TIFS P_3

t / x	t_1	t_2
x_1	$\langle 0.6,0.2 \rangle$	$\langle 0.3,0.3 \rangle$
x_2	$\langle 0.8,0.0 \rangle$	$\langle 0.6,0.2 \rangle$
x_3	$\langle 1.0,0.0 \rangle$	$\langle 0.5,0.4 \rangle$

Suppose that there is a sample P to be recognised and represented by the following TIFS (Table 10):

Table 10. TIFS P

t / x	t_1	t_2
x_1	$\langle 0.5,0.3 \rangle$	$\langle 0.6,0.2 \rangle$
x_2	$\langle 0.6,0.2 \rangle$	$\langle 0.5,0.5 \rangle$
x_3	$\langle 0.8,0.1 \rangle$	$\langle 0.1,0.3 \rangle$

Based on the recognition rule of maximum degree of similarity measures (denoted by e) between the two TIFSs, we can describe the process of assigning P to P_i ($i = 1,2,3$) by the relation

$$e = \arg\max_{1 \le i \le 3} \{C_T(P_i, P).$$

From formula (1), we can compute the cosine similarity between P_i ($i = 1,2,3$) and P as follows:

$$C_T(P_1, P) = 0.9590, \quad C_T(P_2, P) = 0.8264, \quad and \quad C_T(P_3, P) = 0.9243.$$

According to the values of C_T and the recognition rule of maximum degree of similarity between TIFSs, we can classify the unknown pattern P in P_1.

The similarity measures between two TIFSs can be helpful in determining some of the diseases of the patient. In fact, the medical diagnosis is also a pattern recognition problem. If we consider P_1, P_2 and P_3 as a set of diagnoses and x_1, x_2 and x_3 as a set of temporal visible symptoms of these diseases, we try to explore the possibility that a patient with certain temporal visible symptoms P may be suffering from one of the known diseases P_1, P_2 and P_3. If P is significantly similar to P_1, then we conclude that the patient is possibly suffering from the disease P_1.

CONCLUSIONS

We have proposed similarity measures which include time in the TIFS theory in order to recognise the difference between different TIFSs. The proposed similarity measures can deal with spatio-temporal problems in a more effective and reasonable manner. A comparative study has affirmed that the cosine similarity measure is more accurate and attains satisfactory performance on the spatio-temporal pattern recognition.

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