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Full Paper

The value of h/e^2 from quantum Hall effect

Keshav N. Shrivastava

Department of Physics, Faculty of Science, University of Malaya, Kuala Lumpur 50603, Malaysia

E-mail: keshav1001@yahoo.com

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Abstract: The quantum Hall effect and the emergence of the value of h/e^2 is found to be understood within five steps. Here h is the Planck's constant and e is the charge of the electron. The Hall resistivity is found to become a function of spin. For positive spin, one value is found but for negative sign in the spin, another value occurs. In this way, there is never only one value of the resistivity but doubling of values. The value of h/e^2 is a special case of the more general dependence of resistivity on the spin. We investigate the effect of Landau levels. For extreme quantum limit, n=0, the effective charge of the electron becomes (1/2)ge. The fractional charge arises for a finite value of the angular momentum. There is a formation of spin clusters. As the field increases, there is a phase transition from spin ½ to spin 3/2 so that g value becomes 4 and various values of n in Landau levels, g(n+1/2), form plateaus in the Hall resistivity. For finite values of the orbital angular momenta, many fractional charges emerge. The fractional as well as the integral values of the charge are in full agreement with the experimental data. The generalised constant is h/[(1/2)ge]e which under special conditions becomes h/e^2, the ratio of Planck's constant to the square of the electron charge. The flux is usually quantised in units of φ_o =hc/e. When the angular momentum is properly taken into account, hc/e is replaced by hc/(1/2)ge. Thus, we predict a new superfluid which has (1/2)ge in place of the charge, e.

Keywords: Hall effect, constant h/e^2, spin, charge

Introduction

Recently, we have shown that fractional charges occur in the quantum Hall effect and it can be explained by a few steps [1]. The quantum Hall effect is an experimental observation of plateaus in the Hall current which are explained by means of a wave function so that there is a concept of quasiparticles. These quasiparticles may be bosons, fermions or anyons. That is for the theorists to

Maejo Int. J. Sci. Technol. **2010**, *4*(01), 20-32

resolve with or without the use of experimental data. In the constrictions of wires, electron clusters are formed which have spin. It is possible to suggest that repulsive Coulomb interactions give rise to fractional charges as compared with the charge-density waves. Laughlin [2] has suggested a possible wave function which might explain a few fractional charges. Wilczek [3] has the ideas of anyons which obey fractional statistics. It was also suggested that flux quanta may be attached to the electrons, which might explain the symmetries found in the plateaus. Anderson [4] has suggested an alternative to the Laughlin's wave function. Alex Mueller [5] realised the importance of doping in understanding high temperature superconductors, which in turn are important for the understanding of pairing of electrons. We explained the quantum Hall effect with more than 101 plateaus by using the spin [6]. There are six steps in this mesa:

Step 1: Magnetic field. The energy of an electron in a magnetic field is given by,

$$g\mu_B H = \eta\omega_c \qquad (1)$$

where g is the Lande factor, μ_B is the Bohr magneton and c is the velocity of light,

$$\mu_B = \frac{e\eta}{2mc} \qquad (2)$$

Substituting (2) in (1),

$$\frac{1}{2} g \frac{eH}{mc} = \omega_c \qquad (3)$$

Thus, the effective charge of an electron can be written as,

$$e^* = \frac{1}{2} ge\,. \qquad (4)$$

This is an important step because it gives the effective charge of a quasiparticle. When g=2, which is the spin-only value, the effective charge becomes e*= e.

Step 2: Landau levels. The energy levels of an electron in two dimensions look like that of a harmonic oscillator,

$$(n+\frac{1}{2})\eta\omega_c = (n+\frac{1}{2})g\mu_B H\ . \qquad (5)$$

For n = 0,

$$E_o = \frac{1}{2} g\mu_B H = \frac{1}{2}\eta\omega_c \qquad (6)$$

We can remember, just in case we need this energy term with a factor of ½.

Step 3: Hall effect. The classical Hall resistivity is linearly proportional to the magnetic induction. It is used to determine the concentration, *n*, the number of electrons per unit area.

$$\rho = \frac{B}{nec} \qquad (7)$$

The flux within the area, A, is quantised in the units of $\phi_o = hc/e$,

$$B.A = n'\phi_o \qquad (8)$$

Substituting (8) in (7),

$$\rho = \frac{n'\phi_o}{nAec} = \frac{h}{ie^2}\ \text{(i = integer)} \qquad (9)$$

Substituting (4) in (9),

Maejo Int. J. Sci. Technol. **2010**, *4*(01), 20-32

$$\rho = \frac{h}{(\frac{1}{2}ge)e} \tag{10}$$

Note that one e comes from the Hall effect and the other comes from flux quantisation.

Step 4: Fractional values. We define the g values linear in the angular momenta and allow both signs of spin in the total angular momentum, j = $l \pm s$. Then,

$$g = \frac{2j+1}{2l+1} = \frac{2(l \pm s)+1}{2l+1} \tag{11}$$

$$\frac{1}{2}g = \frac{l + \frac{1}{2} \pm s}{2l+1}. \tag{12}$$

For s=1/2, $\frac{1}{2}g_+ = \frac{l+1}{2l+1}, \frac{1}{2}g_- = \frac{l}{2l+1}$, which we tabulate (Table 1):

Table 1. The fractional charge for two signs of s

l	$\frac{1}{2}g_+$	$\frac{1}{2}g_-$
0	1	0
1	2/3	1/3
2	3/5	2/5

The (1/2)g=1 is the correct spin-only value and all of the fractions are correct including the 1/3 charge. These fractional values agree with the experimental data.

Step 5: Spin 3/2. For l = 0, s = 3/2, $\frac{1}{2}g_+ = \frac{l + \frac{1}{2} + \frac{3}{2}}{2l+1} = 2$, $g_+ = 4$. The values of $n + \frac{1}{2}$ are,

½, 3/2, 5/2, 7/2, 9/2, …

and the values of $g_+(n + \frac{1}{2})$ are,

2, 6, 10, 14, 18, …

This series has been observed in the experimental data. Note that if one-particle states are at g/2, then two-particle states occur at g.

Step 6: Comparison.

(i) Equal spin pairing: Balian and Werthamer [7]. Of course these days a better calculation with proper treatment of singlets and triplets is available.

(ii) Zero momentum, spin singlet pairs, k$\uparrow$ and -k$\downarrow$: B.C.S. pairing in the conduction band [8].

(iii) Proton spin triplets, $\uparrow\uparrow$: Leggett [9].

(iv) Our result: Shrivastava [10],

Spin up, $\uparrow$, charge e*=2/3;

Spin down, $\downarrow$, charge e*=1/3.

Maejo Int. J. Sci. Technol. **2010**, *4*(01), 20-32

$$\rho = \frac{h}{(\frac{1}{2} g_{\pm} e)e} . \qquad (13)$$

The theory of fractional charges compares in quality with those of Balian and Werthamer [7], Bardeen et al. [8] and Leggett [9], and explains 101 fractional charges. Hence the flux quantises as B×area=n'hc/[(1/2)$g_{\pm}$e].

The Value of h/e^2

The resistivity at the plateaus is quantised in the units of h/e^2. Usually, the electron is associated with the electromagnetic field, the same way as the charge density is, in the Maxwell equations. The electric and magnetic field vectors are linked to the charge density. However, the charge is defined in such a way that the effect of self electromagnetic fields is already included in the value of the charge,

$$e = 1.602\ 176\ 487(40) \times 10^{-19} \text{ Coulomb} \qquad (14)$$

The Planck's constant is associated with the frequency or the wave length of a particle,

$$h = 6.626\ 068\ \ 96(33) \times 10^{-34} \text{ Js} . \qquad (15)$$

It is a matter of pencil calculation to show that,

$$h/e^2 = 258\ 12.807\ 5651 \text{ Ohm} . \qquad (16)$$

This constant was measured by von Klitzing et al [11]. In their paper, the value given is 25813 Ω. The calculation of h/e^2 does not require that there should be two dimensionality or there should be Landau levels. However, the experimental value requires the Hall geometry. The value of h/e^2 does not require any electrodynamic correction. The fine structure constant is defined in such a way that,

$$h/e^2 = \mu_o c/(2\alpha) \qquad (17)$$

where $\mu_o = 4\pi \times 10^{-7}$ H/m and c is the velocity of light. The above expression is actually an identity because,

$$\alpha = \frac{e^2}{4\pi\varepsilon_o \eta c} \qquad (18)$$

where $\varepsilon_o=1/(\mu_o c^2)$. At the present time, the value of the inverse fine structure constant [12] is 1/α = 137.035 999 084(51), which is another way of writing the value of h/e^2. These are one and the same and not two different quantities. How the accuracy has become so high is another question but in 1965, the value was 137.0388(6). The gyromagnetic ratio of the electron is given by Mohr et al. [12],

$$g/2 = 1.001\ 159\ 652\ 180\ 73(28) \qquad (19)$$

This value is related to the fine structure constant,

$$g/2 = 1 + C_2(\alpha/\pi) . \qquad (20)$$

In this way, g is related to α and α determines h/e^2. However, g is subject to the electrodynamical corrections whereas h/e^2 is not. The electron is associated with the electromagnetic field because of the charge. The electromagnetic field is quantised in terms of photons. Therefore, there are many Feynmann

Maejo Int. J. Sci. Technol. **2010**, *4*(01), 20-32

diagrams which describe the electron-photon interaction so that many more terms arise in (20) which have to be carefully added. The Lande's formula gives,

$$g/2 = 1 \tag{21}$$

for l = 0 and the electrodynamic correction is,

$$\frac{g}{2}\Big|_{electrodynamic} = 0.00115965218073(28) \tag{22}$$

so that,

$$\frac{g}{2} = \frac{g}{2}\Big|_{Lande} + \frac{g}{2}\Big|_{electrodynamic} \ . \tag{23}$$

The g value can be separated into electrodynamic part and Lande's part but in the case of the value of the charge such a separation is not available. The Lande's formula [13] does not contain the electrodynamics but it contains the angular momenta, L, S and J. If there is any correction to the value of the charge due to the electrodynamics, it is already included in the tabulated value of e. There is a problem of gauge invariance as to which h/e is fixed and only one e in h/e^2 is subject to measurement. If both values of e are equal we get the h/e^2. In our theory [14-19] the resistivity is,

$$\rho = \frac{h}{\frac{1}{2}ge^2} \tag{24}$$

where (1/2)g does not include the electrodynamic correction. In fact, such electrodynamic corrections are already included in h/e^2. We use the definition $g=(2j+1)/(2l+1)$ so that for $j = l \pm s$, there are two values of g which we call $g_\pm$,

$$g_\pm = \frac{2(l \pm s)+1}{2l+1} \ . \tag{25}$$

Note that this value of $g_\pm$ does not have the electrodynamic correction. The expression (17) suggests that h/e^2 is equivalent to α and (20) relates α to g value. When l = 0,

$$g_\pm = 2(\pm s)+1. \tag{26}$$

For s=1/2 for + sign, $g_+=2$ so that $(1/2)g_+=1$ and the result (24) gives h/e^2. For s=1/2 and negative sign, $g_-=0$ and we get $\rho \to \infty$, or the conductivity, $\sigma \to 0$. We call these values von Klitzing constants, which now have two values,

$$R_K = h/e^2 \tag{27}$$

and

$$R_K = \infty \ . \tag{28}$$

For l =1, s=1/2 for positive sign, (25) gives,

$$g_+ = \frac{2(1+\frac{1}{2})+1}{3} = \frac{4}{3} \tag{29}$$

or $(1/2)g_+=2/3$, which makes von Klitzing value,

$$R_K = \frac{h}{\frac{2}{3}e^2} \ . \tag{30}$$

For l =1, s=1/2 and negative sign in (25),

Maejo Int. J. Sci. Technol. **2010**, *4*(01), 20-32

$$g_{-}=\frac{2(1-\frac{1}{2})+1}{3}=\frac{2}{3} \qquad (31)$$

or $(1/2)g_{-}=1/3$ so that the von Klitzing resistivity becomes,

$$R_K=\frac{h}{\frac{1}{3}e^2} \qquad (32)$$

In this way many values of the Klitzing constant can be predicted. The fractional values calculated here agree with the measured values of Tsui et al [20].

The Harmonic Oscillator

The eigen values of the harmonic oscillator are given by,

$$E_n=(n+\frac{1}{2})\eta\omega \qquad (33)$$

where,

$$\eta\omega = g\frac{e\eta}{2mc}B. \qquad (34)$$

For n = 0, $E_o=(1/2)\eta\omega$ so that the frequency becomes,

$$E_o=\frac{1}{2}\eta\omega=\frac{1}{2}g\frac{e\eta}{mc}B \qquad (35)$$

This means that we can replace e by (1/2)ge or e*=(1/2)ge. The von Klitzing resistivity now becomes,

$$R_K=\frac{h}{\frac{1}{2}g_{\pm}e^2} \qquad (36)$$

where we can generate a lot of values by changing *l* and s but it is clear that there are pairwise values due to ± and not single value, i.e. there is a doubling of values. From (25) we can calculate the values of $g_{\pm}$ for various values of *l* and s, which gives values of the resistivity. We use the harmonic oscillator-type expression, so that (24) becomes,

$$R_K=\frac{h}{(n+\frac{1}{2})g_{\pm}e^2}\ . \qquad (37)$$

For,

n = 0, 1, 2, 3, 4, 5, 6,

the values of n+(1/2) are,

0, 3/2, 5/2, 7/2, 9/2, …

For S=3/2, *l* =0 we have for the positive sign,

$g_{\pm}$ =2(±3/2) +1 = 4(for + sign). The values of g_{+}(n+1/2) are now,

0, 6, 10, 14, 18, … (38)

This series is actually observed in the experimental data. As we can see, there is no need of random topological numbers, nor of Chern numbers or Hofstadter butterfly [21]. The growth of the series such as that in (25) is not a fractal growth and it does not have a constant chemical length.

Maejo Int. J. Sci. Technol. **2010**, *4*(01), 20-32

The g Value and α

The electron produces its own electromagnetic field which changes the g value. This is a small field but quite noticeable in ordinary electron-spin resonance experiments. The magnetic moment of the electron is,

$$\mu = -\frac{1}{2} g\mu_B \frac{S}{\eta/2} \qquad (39)$$

where S is the spin. Usually S=1/2, but in solid state electron clusters are formed so that it is not limited to ½ and it may be 1, 3/2, 5/2, etc. The accurate value of g/2 is needed to obtain the magnetic moment of the electron. Therefore, it is important to calculate the energy contributions of the electron-photon interaction which can be used to redefine the g value. Thus, an expansion has been considered,

$$\frac{1}{2} g = 1 + c_2(\alpha/\pi) + c_4(\alpha/\pi)^2 + c_6(\alpha/\pi)^3 + c_8(\alpha/\pi)^4 + c_{10}(\alpha/\pi)^5 + ... \qquad (40)$$

in which all of the coefficients have been carefully calculated to find,

$$\alpha^{-1} = 137.035\ 999\ 084(33) \qquad (41)$$

These calculations are limited to l =0, s=1/2 only. Therefore, two values of g are not obtained. Even then there are two values due to the ± in (25). One of these values is zero and the other is 2 besides the electrodynamic correction which is known for l =0. Let us take only 2 terms and substitute 0 and 2 for the g value. Then we obtain two equations,

$$\frac{1}{2}(2 + g_{ed}) = 1 + c_2^{(+)}(\alpha^{(+)}/\pi) \qquad (42a)$$

$$\frac{1}{2}(0 + g_{ed}) = 1 + c_2^{(-)}(\alpha^{(-)}/\pi) \qquad (42b)$$

leaving out small terms. The solution of the second of these gives negative value for $c_2^{(-)}\alpha^{(-)}$, which means that $c_2^{(+)}$ is not equal to $c_2^{(-)}$. Therefore, the values of the coefficients depend on the g values. The sign of the spin is contained in the g value so that both the positive and negative spin values are important.

Two Constants h/e^2 and $h/(g_\pm/2)e^2$

The resistivity at n = 0 in (24) for positive sign of the spin is,

$$\rho = \frac{h}{\frac{1}{2} g_\pm e^2} \qquad (43)$$

where $g_\pm$ must be taken from (25) and is free from the electrodynamic effects. We list some of the values which give the quantisation of the resistivity:

h= 6.626068 960 (330)× 10^{-34} Js,

e=1.602 176 487(40) × 10^{-19} Coulomb,

h/e^2= 25812.807 5651 Ohm {pencil calculation},

(1/2)g=1.001 159 652 180 73(28) [22],

h/e^2= 25812.807557(18) Ohm [12].

By taking only two terms from the right hand side of (40), we find that the charge can be completely eliminated,

Maejo Int. J. Sci. Technol. **2010**, *4*(01), 20-32

$$\rho = \frac{h}{e^2} = \frac{hc_2'}{\pi(4\pi\varepsilon_o \eta c)(\frac{g}{2}-1)} \qquad (44)$$

but the two values of the resistivity are exactly equal. The error in the experimental value of 25812.8 Ω is perhaps not more than ±0.20 Ω. The expression (25) gives the doubling of values due to ± signs and gives the correct fractional values of the charges which agree with the measured values.

Spin and Resistivity

There is a special case when (1/2)g=1,

$$\frac{1}{2}g_{\pm} = \frac{l+\frac{1}{2}\pm s}{2l+1} \qquad (45)$$

which occurs for l =0, s=+1/2. For this case the resistivity (24) is the same as von Klitzing's value. In cases of finite l and s, the physics of the problem is different from that of von Klitzing et al. [11], so that von Klitzing's constant becomes a special case of "spin-dependent" phenomenon [6]. The values of $\rho_{K(+)}$ and $\rho_{K(-)}$ from the expression,

$$\rho_{K(\pm)} = \frac{h}{\frac{1}{2}g_{\pm}e^2} \qquad (46)$$

are given in Table 2 along with the values of $g_{\pm}$. A plot of $\rho_{K(\pm)}$ as a function of l is given in Figure 1.

Table 2. The values of various constants by varying the value of *l*

l	$\rho_{K(+)}$	$\rho_{K(-)}$	(1/2)g_+	(1/2)g_-
0	1	∞	1	0
1	3/2	3/1	2/3	1/3
2	5/3	5/2	3/5	2/5
3	7/4	7/3	4/7	3/7
4	9/5	9/4	5/9	4/9
5	11/6	11/5	6/11	5/11
6	13/7	13/6	7/13	6/13
7	15/8	15/7	8/15	7/15
8	17/9	17/8	9/17	8/17
9	19/10	19/9	10/19	9/19
∞	2	2	1/2	1/2

Maejo Int. J. Sci. Technol. **2010**, *4*(01), 20-32

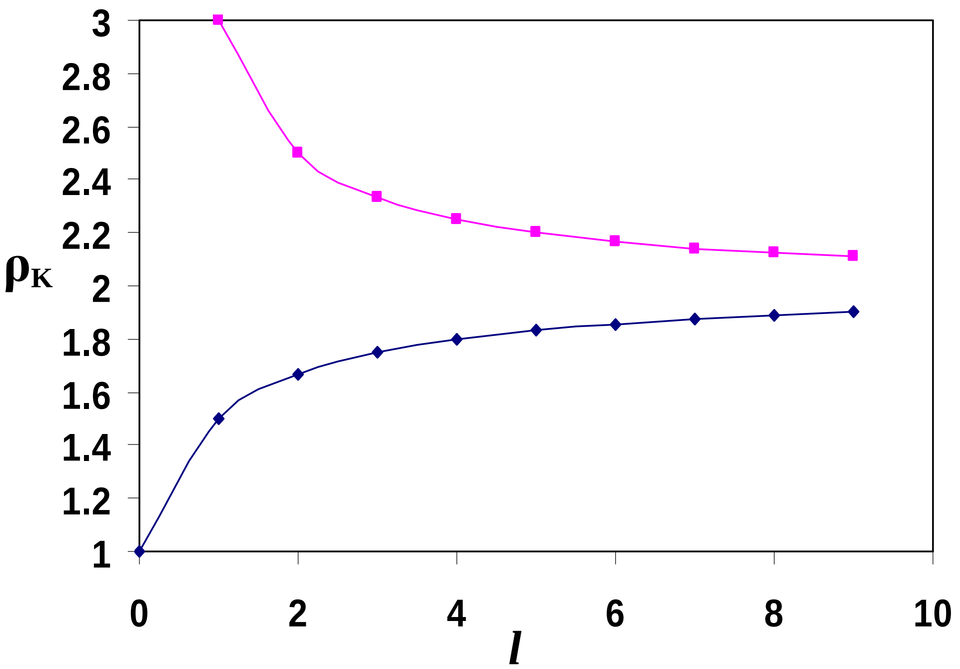

Figure 1. The variation of resistivity as a function of *l*: the upper curve is (-) spin and lower curve is (+) spin.

At l =0, ρ_K=1, we obtain the von Klitzing's constant. Otherwise, there are many values and the von Klitzing constant is a special case of more general constants:

$\rho_{K(+)}$(l =0, s=1/2) = Von Klitzing's constant (47)

$\rho_{K(\pm)}$ ($l \neq 0$, s=±1/2, ± 1, ± 3/2, ±,2, ...) = General constants. (48)

A plot of $\rho_{K(\pm)}$ as a function of (1/2)g from Table 2 is given in Figure 2. When (1/2)g=1, we obtain the von Klitzing's constant, otherwise the more general constants exist.

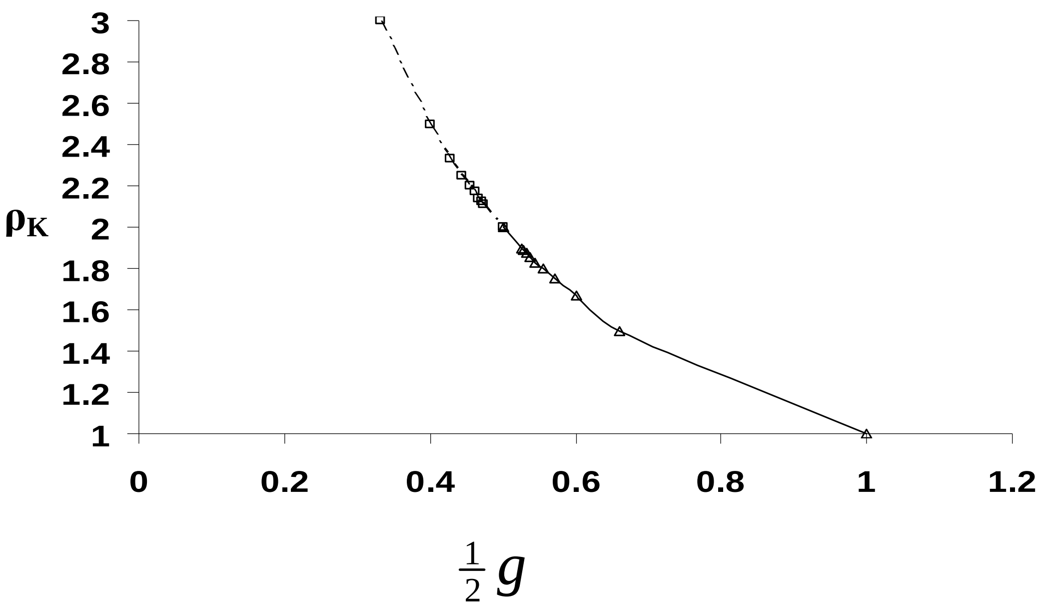

Figure 2. Plot of resistivity as a function of (1/2)g. The continuous line on the right hand side of 0.5 has (+) spin and the broken line on the left hand side of 0.5 has (-) spin.

Maejo Int. J. Sci. Technol. **2010**, *4*(01), 20-32

Turning Points

As the gate voltage is increased, the resistivity starts turning towards the plateau. This phenomenon can occur when spins start turning. When the resistivity is at the Hall effect value away from the plateau region, the electron spin starts turning until the area is so adjusted as to satisfy the flux quantisation, which means that the vortex area becomes an integral multiple of the flux quanta divided by the field area, i.e. $n\varphi_o/B$. The area in the Hall region is infinite. As the spins turn, the area starts reducing from the infinite value to the quantised value.

The change in resistivity from the turning point to the plateau is about 72.7 Ohm, compared with $h/4e^2$=6453.201 Ohm. A plot of the resistivity as a function of gate voltage is given in Figure 3. At the turning point the resistivity is,

$$\rho_{turn} = 6471.21\ \Omega \qquad (49)$$

compared with the pencil calculation of $h/4e^2$= 6453.03 Ω . These two values are off by 18.18 Ω. In order to compare the turning point value with the plateau value, we define,

$$\delta\rho = \rho_{turn} - \rho_{plateau.} \qquad (50)$$

Then the value of $\rho_{plateau}$ is 25812.8075 Ω whereas $\rho_{turn}(i=1)$ is 25884.84 Ω so that,

$$\delta\rho = 72.0\ \Omega. \qquad (51)$$

This, in principle, makes the value of h/ie^2 (i=integer) quite uncertain. The experimental uncertainty in 25812.8 is only 0.2 Ω but the in-principle uncertainty is 2.8×10^{-3}, which is a few parts per thousand. The plateau measurement is obviously much more accurate than the difference between the plateau and the turning point values. In such a case the "in principle" value will play a dominant role. The plateau

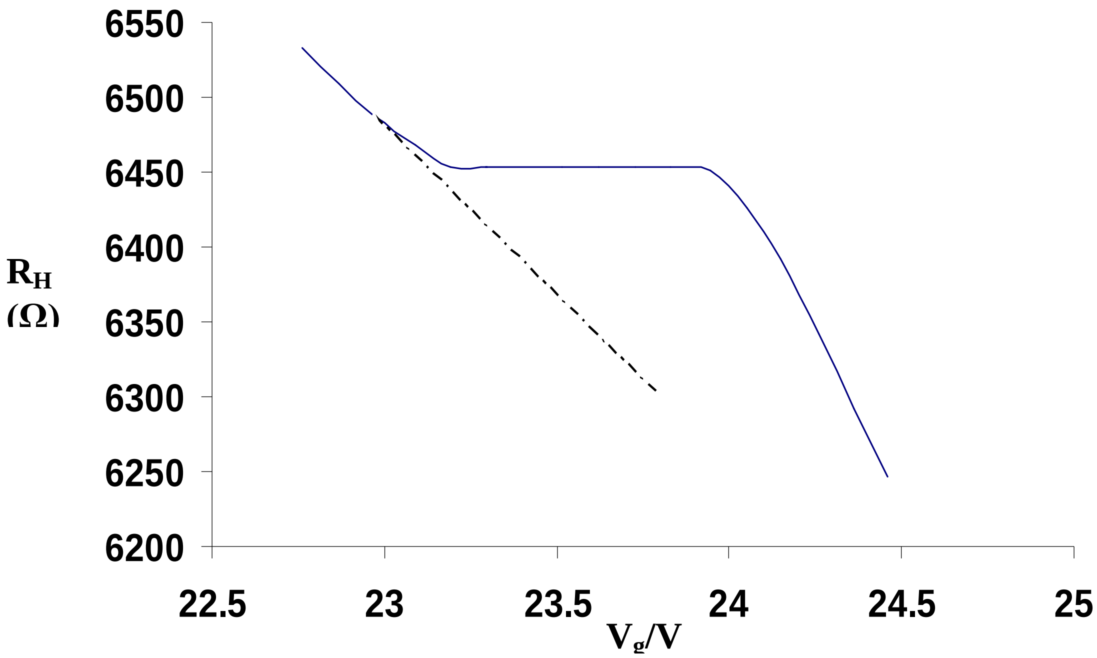

Figure 3. Plot of resistivity as a function of gate voltage. As the gate voltage is increased, the data shows "turning point" before reaching the plateau.

Maejo Int. J. Sci. Technol. **2010**, *4*(01), 20-32

value can be measured up to 8 digits, which means that the accuracy is 1 part in 10^8. If that is the case, the plateau is sharply peaked but the distribution may be extended up to the turning point. It is said that the centre of a line can be located to a large accuracy. That does not mean that there is no line width. The line is an envelope of a large number of events so that there is a finite width. The accuracy of measurement is thus not the accuracy of locating the plateau but the location of the turning point. In Laughlin's work [2] an effort is made to obtain the fractions by correlations. In the present work the fractions arise from the spin. In Laughlin's theory, incompressibility is needed, otherwise the area, A, in the flux quantisation will make the charge flow. The charge can be fractionalised only when A=constant. The flux quantisation condition, B.A=n'hc/e, demands that if the charge has to change, the area A must be a constant. We can check this constancy of the area by applying pressure at the point of the plateaus. In fact, such an experiment has been done, for example by Leadley et al. [23], who recorded the resistivity of $GaAs/Ga_{0.7}Al_{0.3}As$ as a function of pressure and applied magnetic field. It was reported that the dip in the xx-component of the resistivity varies as a function of the applied field. For a pressure of 18.7 kbar the dip at the fractional charge of 1/3 (ν =3) is almost completely wiped off but appears again when the pressure is increased to 20 kbar. In any case, there is some dependence on the pressure so that the area cannot be held constant. In Figure 4 we show the resistivity as a function of pressure, which shows that the system is not incompressible so that the incompressible model of the fractional charge is not necessarily applicable to the present experimental situation. Hence the fractionalisation of the charge is due to spin and the flux is quantised as, B.A= n'φ_o, where φ_o is hc/e which is changed to hc/(1/2)ge, where g=(2j+1)/(2 l + 1). Thus, there is a quantum superfluid in which

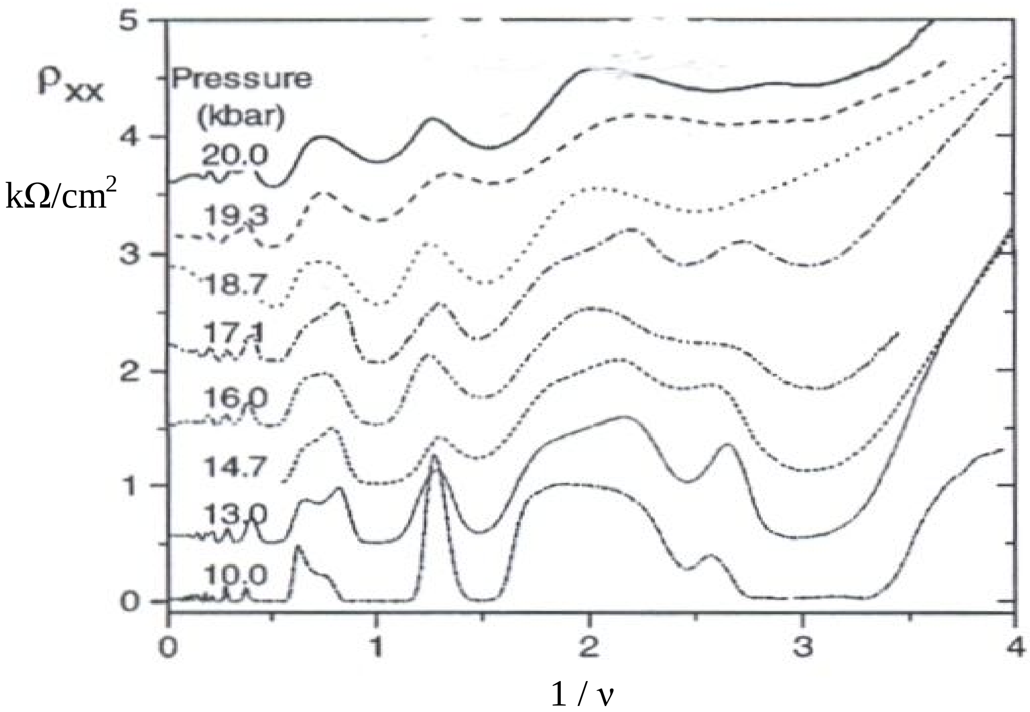

Figure 4. The resistivity of GaAs/AlGaAs as a function of applied pressure. As the pressure is changed from 10.0 kbar to 20.0 kbar, there is considerable change in the behaviour near ν=3, showing that the incompressibility condition is not found [24].

Maejo Int. J. Sci. Technol. **2010**, *4*(01), 20-32

finite angular momenta occur. We have also performed the calculations with Pfaffian determinant from which we find that the non-Abelian determinant is unlikely to correspond to the real material. It is found that the time runs faster in the non-Abelian than in the Abelian wave function [25]. For small matrices such as 2×2, the positive spin works just as good as the negative spin. However, it is interesting to learn that the negative spin plays an important role and it is quite feasible to use it to define the dimensions [26].

Conclusions

The fractional charges occurring in the quantum Hall effect can be explained by the spin properties. It also means that, without spin and by orbital correlations alone, the fractional charge does not occur. If spin is ignored, the correct charge is proportional to the orbital angular momentum as, $E_{\text{effective(spinless)}} \propto \frac{1}{2L+1}$. The von Klitzing's constant, h/e^2, is related to the g value for l = 0 and s =1/2. There is a small correction to this g value due to electrodynamics. Such a correction is already included in the value of the charge of the electron. Another modification to the von Klitzing's constant arises from $g_{\pm}$ which is due to (2j+1)/(2l+1). This effect produces fractional values which agree with the experimental observations. In addition to the von Klitzing's constant, there are more general constants which depend on spin. It is impossible for the von Klitzing constant to occur alone. The constants occur in pairs. For finite values of l and s including the values other than ½, a whole series of constants arise. There is a "turning point" phenomenon. Although the value of h/e^2 is very sharply peaked, it cannot be ignored that there is a distribution. We are able to construct the basic theory which correctly gives the fractional charges in agreement with the data. We find that it is necessary to introduce negative spin and that the charge gets coupled to spin. There is a new condition on the flux quantisation which depends on spin.

References

1. K. N. Shrivastava, "Elementary theory of quantum Hall effect", *Maejo Int. J. Sci. Technol.*, **2008**, *2*, 285-297.
2. R. B. Laughlin, "Anomalous quantum Hall effect: An incompressible liquid with fractionally charged excitations", *Phys. Rev. Lett.*, **1983**, *50*, 1395-1398.
3. F. Wilczek, "Magnetic flux, angular momentum and statictics", *Phys. Rev. Lett.*, **1982**, *48*, 1144-1146.
4. P. W. Anderson, "Remarks on the Laughlin theory of fractionally quantized Hall effect", *Phys. Rev. B*, **1983**, *28*, 2264-2265.
5. K. A. Mueller, "On the superconductivity in hole doped cuprates", *J. Phys. Condens. Matter*, **2007**, *19*, 251002-1-13.
6. K. N. Shrivastava, "Interpretation of fractions in quantum Hall effect: Wei Pan's data", *Amer. Inst. Phys. Conf. Proc. (USA)*, **2009**, *1150*, 59-67.
7. R. Balian and N. R. Werthamer, "Superconductivity with pairs in a relative p wave", *Phys. Rev.*, **1963**, *131*, 1553-1564.

Maejo Int. J. Sci. Technol. **2010**, *4*(01), 20-32

8. J. Bardeen, L. Cooper and J. R. Schrieffer, "Theory of superconductivity", *Phys. Rev.*, **1957**, *108*, 1175-1204.
9. A. J. Leggett, "Interpretation of recent results on He3 below 3mK: A new liquid phase", *Phys. Rev. Lett.*, **1972**, *29*, 1227-1230.
10. K. N. Shrivastava, "Rational numbers of the fractionally quantized new states and their observation in fractionally quantized Hall effect", *Phys. Lett. A*, **1986,** *113*, 435-436; *115*, 459(E).
11. K. von Klitzing , G. Dorda and M. Pepper , "New method for high accuracy determination of the fine structure constant based on quantized Hall resistance", *Phys. Rev. Lett.*, **1980,** *45*, 494-497.
12. P. J. Mohr, B. N. Taylor and D. B. Newell, "The fundamental physical constants", *Phys. Today*, **2007**, *60*, 52-54.
13. A. Lande, "Quantum Mechanics", Cambridge University Press, Cambridge, **1951**.
14. K. N. Shrivastava, "Additional Dirac matrix in quantum Hall effect", *Amer. Inst. Phys. Conf. Proc. (USA)*, **2008**, *1017*, 47-56.
15. K. N. Shrivastava, "Energy band crossing points in multilayers of graphene", *Amer. Inst. Phys. Conf. Proc. (USA)*, **2008**, *1017*, 326-330.
16. K. N. Shrivastava, "The theory of the quantum Hall effect", *Amer. Inst. Phys. Conf. Proc (USA)*, **2008**, *1017*, 422-428.
17. K. N. Shrivastava, "Infrared of thin film graphene in a magnetic field and the Hall effect", *Proc. SPIE (USA)*, **2008**, *7155*, 71552F-1-8.
18. A. N. Rosli and K. N. Shrivastava, "Von Klitzing's constant as a special case of generalized constants", *Amer. Inst. Phys. Conf. Proc. (USA)*, **2009**, *1136*, 469-473.
19. K. N. Shrivastava, "The fractional quantum Hall effect: The cases of 5/2 and 12/5", *Amer. Inst. Phys. Conf. Proc. (USA)*, **2009**, *1169*, 48-54.
20. D. C. Tsui, H. L. Stormer and A. C. Gossard, "Two dimensional magnetotransport in the extreme quantum limit", *Phys. Rev. Lett.*, **1982**, *48*, 1559-1562.
21. D. R. Hofstadter, "Energy levels and wave functions of Bloch electrons in rational and irrational magnetic fields", *Phys. Rev. B*, **1976**, *14*, 2239- 2249.
22. D. Hanneke, S. Fogwell and G. Gabrielse , "New measurement of the electron magnetic moment and the fine structure constant", *Phys. Rev. Lett.*, **2008**, *100*, 120801-1-4.
23. D. R. Leadley, R. J. Nicholas, D. K. Maude, A. N. Utjuzh, J. C. Portal and C. T. Foxon,"Fractional quantum Hall effect measurements at zero g factor", *Phys. Rev. Lett.*, **1997**, *79*, 4246-4249.
24. K. N. Shrivastava, "The derivation of the von Klitzing's constant", *Natl. Acad. Sci. Lett. (India)*, **2009**, *32*, 243-249.
25. K. N. Shrivastava, "Pfaffian states: Quantum computation", *Amer. Inst. Phys. Conf. Proc. (USA)*, **2009**, *1169*, 241-246.
26. K. N. Shrivastava, "Negative-spin quasiparticles in quantum Hall effect", *Phys. Lett. A*, **2004**, *326*, 469-472.