Maejo Int. J. Sci. Technol. **2010**, *4*(02), 235-249

Maejo International
Journal of Science and Technology

ISSN 1905-7873
Available online at www.mijst.mju.ac.th

Review

Mathematical transforms and image compression: A review

Satish K. Singh[1,*] and Shishir Kumar [2]

[1] Department of Electronics and Communication Engineering, Jaypee University of Engineering and Technology, Guna 473 226, India
[2] Department of Computer Science and Engineering, Jaypee University of Engineering and Technology, Guna 473 226, India

* Corresponding author, e-mail: satish432002@rediffmail.com

Received: 15 January 2010 / Accepted: 14 July 2010 / Published: 15 July 2010

Abstract: It is well known that images, often used in a variety of computer and other scientific and engineering applications, are difficult to store and transmit due to their sizes. One possible solution to overcome this problem is to use an efficient digital image compression technique where an image is viewed as a matrix and then the operations are performed on the matrix. All the contemporary digital image compression systems use various mathematical transforms for compression. The compression performance is closely related to the performance by these mathematical transforms in terms of energy compaction and spatial frequency isolation by exploiting inter-pixel redundancies present in the image data. Through this paper, a comprehensive literature survey has been carried out and the pros and cons of various transform-based image compression models have also been discussed.

Keywords: image transforms, compression, entropy, coding gain, truncation error, quantisation error

---

Introduction

All practical-purpose images are a collection of some structured data generating some degree of correlation between neighbouring pixels. Correlation is closely related to redundancy which is known as inter-pixel redundancy. It requires a reversible transform to remove the inter-pixel redundancy by decorrelating the image in a more compact manner [1-2]. Thus any image having the correlated pixels

Maejo Int. J. Sci. Technol. **2010**, *4*(02), 235-249

can be compressed using transform coding methods where the transform coefficients are highly decorrelated. An image transform can achieve a compression if the numbers of non-zero transform coefficients are smaller on average than the original pixels or data points. After quantisation of the transform coefficients lossy compression can be achieved [3]. An image transform aiming for compression should follow two properties : (a) inter-pixel redundancy minimisation; and (b) spatial frequency isolation.

In digital images the spatial frequencies are important as the low-frequency components correspond to important image features and the high-frequency ones to image details. High frequencies are a less important part of the images and can be quantised more heavily than low frequency coefficients to achieve low-bit rates. Also, the image transforms should be fast and simple giving a choice for linear transformations [3-9].

A linear transformation matrix $[W]$, whose transpose $[W]^T$ will rotate the data matrix X to produce a diagonal covariance matrix for the transformed variable Y where $X = [x_1, x_2, x_3, ..., x_N]^T$ is a vector having N pixel or data points. Then,

$$Y = [W]^T X \tag{1}$$

Each column vector w_i of $[W]$ is a basis vector of new space. So alternatively each element y_i of Y is calculated as

$$y_i = w_i^T X \tag{2}$$

For simple rotation with no scaling, the matrix $[W]$ must be orthogonal, that is

$$[W]^T [W] = I = [W][W]^T \tag{3}$$

where I is the identity matrix. This means the column vectors of matrix $[W]$ are mutually orthogonal and are of unit norm. From equation (3) it is clear that the inverse of an orthogonal matrix is simply its transpose :

$$[W]^T = [W]^{-1} \tag{4}$$

The inverse transformation is calculated as

$$X = [W]Y \tag{5}$$

The total energy after transformation is given as follows:

Maejo Int. J. Sci. Technol. **2010**, *4*(02), 235-249

$$\begin{aligned} \|Y\|^2 &= Y^T Y \\ &= \left([W]^T X\right)^T \left([W]^T X\right) \\ &= X^T [W][W]^T X \\ &= \|X\|^2 \end{aligned} \tag{6}$$

where $\|X\|^2$ is the norm of vector X, defined as follows:

$$\|X\| = \sqrt{X^T X} = \sqrt{\sum_{i=1}^{N} x_i^2} \tag{7}$$

Factors Affecting the Performance of Image Transforms Used for Compression

There are several factors such as entropy, coding gain, quantisation error, truncation error and block size which affect the compression performance of transform-based image compression systems [10].

Entropy

Entropy is a useful means of determining the performance of compression [7-8] and theoretically gives a lower bound on the average number of bits required for encoding without introducing error [10]. The probability of any real-value sample may be zero, causing discrete entropy to be undefined. To cure the problem of undefined discrete entropy, the differential entropy is used as generalised measure for the distribution of information. The differential entropy is given as follows [7]:

$$h(x) = -\int_{-\infty}^{\infty} p(s)\log[p(s)]ds \tag{8}$$

where $h(x)$, s and $p(s)$ are the entropy, samples of a sample space x and probability of samples respectively. For simple distribution such as Gaussian, Uniform and Laplacian, the differential entropy is given as follows:

$$h(x) = \frac{1}{2}\log(\sigma_x^2 + k) \tag{9}$$

where σ_x^2 is the variance of random variable and k is the constant which depends on the data or random-variable distribution.

From equations (8) and (9) it is evident that the image transformation should minimise the sum of differential entropy or the product of variances of the coefficients due to logarithmic terms [8]. The total energy is preserved after transformation due to orthonormality, hence the fixed sum of the coefficient variance [7, 10].

Maejo Int. J. Sci. Technol. **2010**, *4*(02), 235-249

Coding gain

Coding gain is a measure of the compression efficiency of transformation and is given as follows [9-10]:

$$G_w = \frac{\frac{1}{N}\sum_{i=1}^{N}\sigma_{y_i}^2}{\left(\prod_{i=1}^{N}\sigma_{y_i}^2\right)^{1/N}} \tag{10}$$

where the numerator is the algebraic mean of variances which is transform-independent and the denominator is the geometric mean of variances and is transform-dependent. For any arbitrary signal or data, all the variances are almost equal giving a coding gain of 1. For a given energy signal, minimising the product of variances maximises the coding gain and minimises the lower bound on the number of bits required [10, 12].

Quantisation

Quantisation error also plays a very important role in the compression system and should be very low after transformation. Let $\hat{Y}$ be a set of quantised coefficients for a block of data. The reconstructed data is then given as:

$$\hat{X} = [W]\hat{Y} \tag{11}$$

The square error for such block of data is given as follows:

$$\begin{aligned} e^2 &= \left\|\hat{X} - X\right\|^2 = \left(\hat{X} - X\right)^T \left(\hat{X} - X\right) \\ &= \left([W]\hat{Y} - [W]Y\right)^T \left([W]\hat{Y} - [W]Y\right) \\ &= \left(\hat{Y} - Y\right)^T [W]^T[W]\left(\hat{Y} - Y\right) \\ &= \left(\hat{Y} - Y\right)^T \left(\hat{Y} - Y\right) \end{aligned}$$

$$\Rightarrow e^2 = \left\|\hat{Y} - Y\right\|^2 \tag{12}$$

From equation (12), it is clearly visible that for any linear orthogonal transformation having orthonormal vectors, the squared error on reconstruction is the same as that of the coefficients [10].

Truncation error

Another method of reducing the data is to remove some transformed coefficients completely leaving only M out of N coefficients. The truncation error is given in equation (13):

Maejo Int. J. Sci. Technol. **2010**, *4*(02), 235-249

$$E(e^2) = E\left[\frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2\right]$$
$$= \frac{1}{N}E\left[\sum_{i=1}^{M}(y_i - y_i)^2 + \sum_{i=M+1}^{N}(y_i - 0)^2\right]$$
$$= \frac{1}{N}E\left[\sum_{i=M+1}^{N} y_i^2\right]$$
$$\Rightarrow E(e^2) = \frac{1}{N}\sum_{i=M+1}^{N}\sigma_i^2 \tag{13}$$

where E, y_i, $\hat{y}_i$ and σ_i^2 are expected and original values of transformed coefficients, quantised transform coefficients and variances of the transformed coefficients respectively.

If the variances for the truncated coefficients are smaller and smaller, then the truncation error can be minimised [1, 4, 11].

Block size

The linear orthogonal transforms having orthonormal vectors are applied on some block of data to be transformed. The larger the block size is, the greater the decorrelation becomes, hence the greater coding gain [5, 7]. The number of arithmetic operations increases linearly as the block size increases, hence the complexity. Also, the block-based image transform reduces the inter-pixel redundancy among the pixels or data points within the block, leaving no assurance to remove the inter-block redundancy [9, 13].

Types of Image Transforms Used for Compression

Karhunen-Loeve transform and image compression

The Karhunen-Loeve transform (KLT) is a linearly reversible, orthogonal transformation which accomplishes the removal of redundancy by decorrelating the data block elements and is defined by Eigen values of covariance matrix [14]. Hotelling in 1933 [9] developed a method of principal components for removing the correlation from discrete random variable. A continuous version of Hotelling's transform was developed by Karhunen and Loeve in 1960's [15]. KLT is also known as Hotelling Transform or PCA (principal component analysis) [15-16]. The covariance matrix of an arbitrary data block is real and symmetric so the real Eigen values and corresponding Eigen vectors can be found easily. The diagonal covariance matrix $[C]_Y$ of a transformed variable Y is given as:

$$[C]_Y = \begin{bmatrix} \lambda_1 & 0 & . & 0 \\ 0 & \lambda_2 & . & 0 \\ . & . & . & 0 \\ 0 & 0 & 0 & \lambda_N \end{bmatrix} \tag{14}$$

Maejo Int. J. Sci. Technol. **2010**, *4*(02), 235-249

where λ_1, λ_2 .... λ_N are variances of transformed data Y. The diagonal matrix can be calculated from the original covariance matrix $[C]_X$ as follows:

$$[C]_Y = E[YY^T]$$
$$=E\left[\left([W]^T X\right)\left([W]^T X\right)^T\right]$$
$$=E\left[[W]^T\left(XX^T\right)[W]\right]$$
$$[C]_Y = \left[W^T\right][C]_X[W] \tag{15}$$

The column vector of *W* are found as

$$[C]_X w_i^T = \lambda_i w_i \tag{16}$$

where λ_i and w_i are Eigen value and Eigen vector pairs for $i = 1,2,3.......N$. The orthonormal Eigen vectors are found by using Gram-Schmidt orthonormalisation process [10]. KLT minimises the geometric mean of the variance of transform coefficients, thus providing largest coding gain [17]. The basis vectors of KLT are calculated from the original image pixels and are therefore data-dependent. In practical applications these vectors should also be included in the compressed bit streams, making this transform less ideal for practical applications of image compression [12, 18].

Discrete cosine transform and image compression

Discrete cosine transform (DCT) [19] is very important for compression. DCT is a discrete time version of Fourier-cosine series and can be computed with fast-Fourier-transform-like algorithms. Unlike discrete Fourier transform, DCT is a real value and provides a better approximation of a signal with fewer transform coefficients [20].

The DCT of a discrete signal X is given as

$$Y(f) = \sqrt{\frac{2}{N}} C_f \sum_{t=0}^{N-1} X(t) Cos\left[\frac{(2t+1) f\pi}{2N}\right] \tag{17.a}$$

$$C_f = \frac{1}{\sqrt{2}} \quad : f = 0$$
$$= 1 \quad : f \neq 0, f = 1,2,3........N-1 \tag{17.b}$$

$$\Rightarrow Y = [Y(0), Y(1),........Y(N-1)] \tag{17.c}$$

where t, f, N and $Y(0)$ are time, frequency, number of points and DC coefficient respectively. $Y(1),........to\ Y(N-1)$ are the AC coefficients and frequency increases as we go from $Y(1),........to\ Y(N-1)$. The inverse DCT transform is given as

Maejo Int. J. Sci. Technol. **2010**, *4*(02), 235-249

$$X(t)=\sqrt{\frac{2}{N}}\sum_{j=0}^{N-1}C_jY(f)Cos\left[\frac{(2t+1)j\pi}{2N}\right] \tag{17.d}$$

where $t=0,1,2,........N-1$ and C_j is the j^{th} component in frequency domain for *j*=0,1,...*N-1*, which is similar to C_f in time domain. The 2-dimensional discrete cosine transform (2D-DCT) and 2-dimensional inverse discrete cosine transform (2D-IDCT) for $M\times N$ matrix are given in equations (18.a) and (18.b) respectively:

$$Y(i,j)=\frac{2}{\sqrt{MN}}C_iC_j\sum_{i=0}^{N-1}\sum_{j=0}^{M-1}X(x,y)Cos\left[\frac{(2t+1)j\pi}{2M}\right]Cos\left[\frac{(2t+1)i\pi}{2N}\right] \tag{18.a}$$

where $0\le i\le N-1$; $0\le j\le M-1$ and $Y(0,0)$ is DC coefficient and $Y(i,j):(i\ne j)\ne 0$ are AC coefficients.

$$X(x,y)=\frac{2}{\sqrt{MN}}\sum_{i=0}^{N-1}\sum_{j=0}^{M-1}C_iC_jY(i,j)Cos\left[\frac{(2y+1)j\pi}{2M}\right]Cos\left[\frac{(2x+1)i\pi}{2N}\right] \tag{18.b}$$

where $0\le i\le N-1$ and $0\le j\le M-1$.

The DCT has as good energy compaction as KLT [12]. The advantage of DCT over KLT is that the former uses a fixed basis which is independent of data or signal. Also, DCT is a block-based transform so performance and complexity is compromised with the block size [19-20].

Discrete sine transform and image compression

Discrete sine transform (DST) is a complementary transform of DCT. DCT is an approximation of KLT for large correlation coefficients whereas DST performs close to optimum KLT in terms of energy compaction for small correlation coefficients. DST is used as low-rate image and audio coding and in compression applications [21-22].

Discrete Walsh-Hadamard transform and image compression

The discrete Wlash-Hadamard transform (DWHT) is the simplest transform to be implemented for any application and is a rearrangement of discrete Hadamard transform matrix [23]. The amount of energy compaction efficiency of DWHT is poorer than that of DCT or KLT so it does not have a potential to be used for data compression [12, 23].

Discrete wavelet transform and image compression

All the linear orthogonal transformations, i.e. KLT, DST and DCT, are blocked transformations which remove the correlation among the pixels or data points inside the block. These transforms do not take care of correlation across the block boundaries [24]. The blocking artifacts are dominating at low bit rates. The blocking effect can be reduced by Lapped orthogonal transforms (LOT) but at the cost of increased computational complexity [25]. A wavelet transform does not require blocking of signal or data points before transformation, resulting in removal of blocking artifacts even at very low bit rates.

Maejo Int. J. Sci. Technol. **2010**, *4*(02), 235-249

Also, wavelet-based subband coding is robust under decoding error and has a good compatibility with human visual system [26]. There are several ways to decompose a signal into various subbands using the wavelet transform, such as octave, adaptive and packet decompositions [27-29]. The octave decomposition is the most used decomposition technique, which non-uniformly splits the bands, rendering the lower frequency part narrower and narrower while leaving out any further decomposition of higher frequency coefficients. Figures 1-3 show a 1-level 2D-wavelet transform (DWT) [29]. Over the past few years many improvements of wavelet-based coding have been developed such as EZW, SPIHT, EBCOT, EPWIC, SFQ, CREW, SR, second generation wavelet coding, wavelet packet image coding, wavelet packet with VQ, and integer wavelet transform coding [30-40].

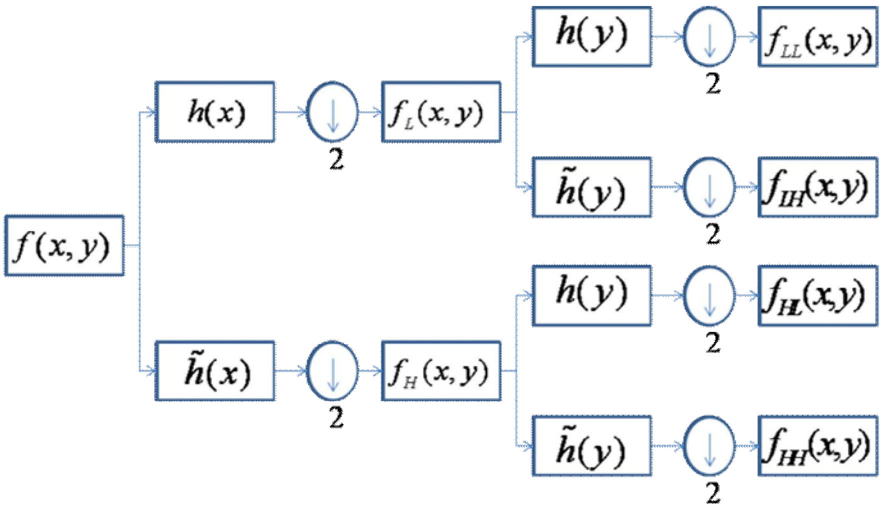

Figure 1. 1-Level 2D-wavelet decomposition: $h(x)$ and $\tilde{h}(x)$ are horizontal low pass and high pass filter functions whereas $h(y)$ and $\tilde{h}(y)$ are vertical low pass and high pass filter functions respectively; $f_L(x,y)$ and $f_H(x,y)$ are horizontal low pass and high pass wavelet coefficients respectively; approximation, horizontal, vertical and diagonal details are respectively represented by $f_{LL}(x,y)$, $f_{LH}(x,y)$, $f_{HL}(x,y)$ and $f_{HH}(x,y)$.

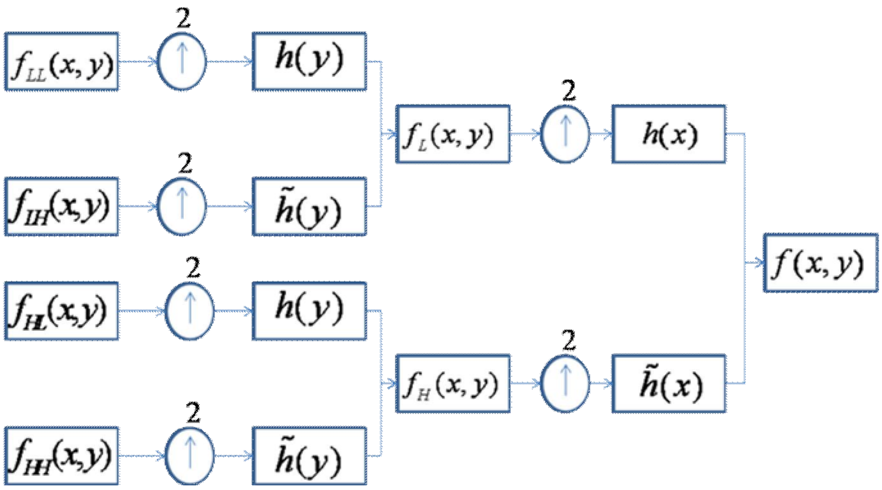

Figure 2. 1-Level 2D-wavelet recomposition: $h(x)$ and $\tilde{h}(x)$ are horizontal low pass and high pass filter functions whereas $h(y)$ and $\tilde{h}(y)$ are vertical low pass and high pass filter functions respectively; $f_L(x,y)$ and $f_H(x,y)$ are horizontal low pass and high pass wavelet coefficients respectively; approximation, horizontal, vertical and diagonal details are respectively represented by $f_{LL}(x,y)$, $f_{LH}(x,y)$, $f_{HL}(x,y)$ and $f_{HH}(x,y)$.

Maejo Int. J. Sci. Technol. **2010**, *4*(02), 235-249

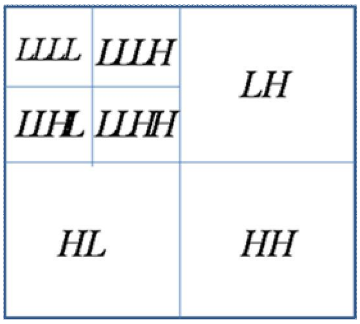

Figure 3. Spectral decomposition and ordering of wavelet coefficients: *L* and *H* are the low pass and high pass wavelet transformed coefficients respectively.

Fractional discrete transform and image compression

In 1929 Wiener [41] introduced a concept of fractional transforms, which led to the development of fractional Fourier transform (FrFT) first developed in 1980 [42]. Almeida [43] explored the time-frequency localisation property and provided a possible application of FrFT in image compression. In the case of fractional transform, one extra free parameter is also there besides time and frequency. In 2000 Gerek and Erden proposed a discrete fractional cosine transform by taking an advantage of the relation between DCT and DFT [44], which was similar to the method of finding DFrFT by Ozaktas et al. in 1996 [45]. In 2005 Singh and Saxena [46] explored the possible application of DFrCT and DFrFT in image compression. The compression performance of fractional transforms depends on the value of free parameter. However, any direct relation between free parameter and compression performance has not been reported. Hence, it is impractical to optimise the free parameter, which results in a recursive and a very slow process for image compression.

Directional discrete transform and image compression

All the transforms as discussed above are 2D transforms implemented by using 1D separable architectures and are not suitable to preserve the image features with arbitrary orientation that is neither vertical nor horizontal [47]. In these cases, they result in large-magnitude high-frequency coefficients. At low bit rates, the quantisation noise from these coefficients is clearly visible, in particular causing annoying Gibbs artifacts at image edges with arbitrary directions. Some work on wavelet and subband transform to incorporate directional information into transforms has been reported. The lifting structure developed by Sweldens provides a good way to incorporate directional information into the wavelet transform [47-49]. Zeng and Fu [50] are the first authors to propose how to incorporate directional information into DCT. Their directional DCT is motivated by SA-DCT (shape-adaptive DCT). Hao et al. [51] proposed a lifting-based directional DCT-like transform for image coding and used it for image compression. The main problem with directional transforms is the selection of optimum direction.

Maejo Int. J. Sci. Technol. **2010**, *4*(02), 235-249

Singular-value decomposition and image compression

Image transform is a very important part of image compression. The optimum transform coder which minimises the mean square distortion of the reproduced data for a given bit rate is the KLT [9]. Other transforms investigated for image or picture compression include DCT, piecewise Fourier Transform, slant transform, linear transform with block quantisation and Hadamard transform [53-57]. Though the energy compaction efficiency of the KLT is very suitable for compression, it is not used in real applications due to its computational complexity [53, 59-60]. Singular-value decomposition-based transformation has an optimal energy-compaction property making it the most appropriate for compression in spite of computation complexity [61]. In the case of singular-value decomposition (SVD) the singular values are image-dependent and must therefore be coded with the associated singular vectors as side information [62]. The optimal energy compaction property was exploited and utilised by McGoldrick et al. [62] and Yang and Lu [63]. McGoldrick et al. calculated singular values as well as singular vectors and the latter were coded by variable-rate vector quantiser. JPEG image coder based on DCT was superior to SVD-based method. Yang and Lu also used SVD in conjunction with vector quantisation giving a superior method by reducing the computational complexity to that of DCT-based method. However, with the application of fast DCT algorithm, this was not a preferred technique [64]. Waldemar and Ramstad [65-66] proposed hybrid KLT-SVD image compression using transform adaptation technique exploiting the local variation of images. This hybrid method was better than KLT-based methods in terms of energy compaction but could not be sustained due to a large number of vectors to be coded. In 2000 Chen [67] used rank approximation method for SVD-based lossy image compression. In rank approximation for SVD-based image compression an image of size $N \times N$ was transformed by SVD to obtain matrices U_{NxN}, S_{NxN} and V_{NxN}, where S is a diagonal $N \times N$ matrix whose number of non-zero diagonal elements determines the rank "k" of the original matrix where $k \le N$. In this method a smaller rank is used to approximate the original image. The total storage space required to restore the original approximated image is $2Nk + k$, where $k \le N$. In order to achieve the goal of compression, used rank should be as follows:

$$k \le \frac{N \times N}{(1+2N)} \tag{19}$$

So by rank approximation method there is a restriction on reconstructed image quality for compressed image. Arnold and McInnes in 2000 [68] reported block-based adaptive rank approximation method similar to most of the popular image compression methods, to exploit the uneven complexity and correlation of image. The work reported by them was based on singular-value distribution of different subblocks in which higher ranks were used for complex subbands. Also, for the same storage space, smaller block sizes of subblocks produced better results [68-69]. Arnold and McInnes further reduced rank of the blocks by rank-one update, in which the respective mean was subtracted from all the elements of the blocks and then SVD and adaptive rank approximation was used. Dapena and Ahalt [69] and Wongsawat et al. [70] reported hybrid DCT-SVD and modified hybrid DCT-SVD image coding algorithms in 2002 and 2004 respectively. Both methods were based on an adaptive selection of block transforms to be used on the basis of complexity and correlation of different blocks. For high correlation, SVD was used while for the rest, DWT was used. In 2003 a hybrid DWT-

Maejo Int. J. Sci. Technol. **2010**, *4*(02), 235-249

SVD-based image coding, which is also a block-based method, was reported by Ochoa and Rao [71-72], who used a criterion of threshold standard deviation for all blocks of Y component to determine whether DWT or SVD has to be used for any particular block. If standard deviation is high, rank-one update is used for that block, otherwise DWT method is used. Ochoa and Rao further extended this method for colour image compression also [73-74]. In 2007 Ranade et al. [75] proposed a modified SVD image compression based on SSVD (shuffled SVD). In this work the block-based shuffling operator was used to get subblocks. The performance of SSVD was shown to be better than SVD in terms of space for the same quality but involved more complex operations. Also, the performance was not even near to DCT-based coding systems. Aase et al. [76] gave a critique on SVD-based image compression and pointed out the major drawback of using lossless SVD transform for image compression. According to them, the singular vectors along with the singular values are stored for lossless reconstruction, which requires $2(1+1/N)$ times more space for $N \times N$ image.

Conclusions

On the basis of the above discussion it can be concluded that any image transform applied for image compression will have minimum entropy, maximum coding gain, minimum quantisation error, minimum truncation error, and moderate block size. Although the KLT shows highest energy compaction, it is a very complex transform and usually takes unfeasible time delay during the transformation. DCT shows as good performance as KLT though the advantage of DCT over KLT is that the former employs fixed basis which is independent of data or signal. Also, DCT is a block-based transform so performance and complexity is compromised with the block size. Another advantage of DCT is its blocking effect for low bit rate applications. DST is also a block-based transform and can be used only for the image or data which have very small correlation. DWHT is very simple to implement but has a very poor performance in terms of energy compaction efficiency. The compaction efficiency of DWT is not very good compared to that of DCT but it can provide a satisfactory performance for the entire range of bit rates. The blocking effect as shown in DCT is removed in the case of DWT as it is a global transform and not the block-based transform. The compression performance of fractional transforms depends on the value of free parameter and it is impractical to optimise the free parameter due to a recursive and very slow process, which is not favourable for compression. Directional discrete transforms are used at low bit rates when the quantisation noise from the transform coefficients is clearly visible, in particular causing Gibbs artifacts at the image edges with arbitrary directions. The optimisation of direction makes it unsuitable for compression. SVD transform has an optimum energy compaction property but needs the requirement of more storage space for lossless compression and has a high level of complexity if it is used globally.

References

1. R. J. Clark, “Transform Coding of Images”, Academic Press, San Diego, CA, **1985**.
2. R. D. Dony and S. Haykins, “Optimally adaptive transform coding”, *IEEE Trans. Image Process.*, **1995**, *4*, 1358-1370.

3. R. D. Dony, S. Haykins C. Coblentz and C. Nahmias, "Compression of digital chest radiographs using a mixture of principle components neural network: An evaluation performance", *Radio Graphics,* **1992**, *16,* 1481-1488.
4. C. Chatterjee, V. P. Roychowdhury and E. K. P. Chong, "On relative convergence properties of principle component analysis algorithm", *IEEE Trans. Neural Networks,* **1998**, *9*, 319-329.
5. R. J. Clark, "Digital Compression of Still Images and Video", Academic Press, San Diego, CA, **1995**.
6. T. Chen, Y. Hua and W. Y. Wan, "Global convergence of Oja's subspace algorithm for principle component extraction", *IEEE Trans. Neural Networks,* **1998**, *9*, 58-67.
7. R. M. Gray, "Source Coding Theory", Kluwer Academic Publishers, Norwell, MA, **1990**.
8. T. Berger, "Rate Distortion Theory", Prentice-Hall, Englewood Cliffs, NJ, **1971**.
9. H. Hotelling, "Analysis of complex of statistical variables into principal components", *J. Educ. Psychol.*, **1933**, *24,* 417-447.
10. K. R. Rao and P. C. Yip, "The Transform and Data Compression Handbook", CRC Press, New York, **2001**.
11. K. I. Diamantaras and S. Y. Kung, "Principal Component Neural Networks: Theory and Applications", John Wiley and Sons, New York, **1996**.
12. D. Solomon, "Data Compression: The Complete Reference", Springer Verlag, New York, **2004**.
13. I. Jolliffe, "Principal Component Analysis", Springer Verlag, New York, **1986**.
14. N Ahmed and K. R. Rao, "Orthogonal Transforms for Digital Signal Processing", Springer Verlag, New York, **1975**.
15. K. Karhunen (translated by I. Selin), "On linear methods in probability theory", Doc. T-131, RAND Corporation, Santa Monica, CA, **1960**.
16. E. Oja and J. Karhunen, "On stochastic approximation of eigen vectors and eigen values of the expectation of a random matrix", *J. Math. Anal. Appl.*, **1985**, *106,* 61-68.
17. N. S. Jayant and P. Noll, "Digital Coding of Waveforms", Prentice-Hall, Englewood Cliffs, NJ, **1984**.
18. V. Solo and X. Kong, "Performance analysis of adaptive eigenanalysis algorithms", *IEEE Trans. Signal Process.*, **1998**, *46*, 636-645.
19. N. Ahmed, T. Natarajan and K. R. Rao, "Discrete cosine transform", *IEEE Trans. Comm.*, **1974**, *C-23,* 90-93.
20. K. R. Rao and P. Yip, "Discrete Cosine Transform: Algorithms Advantages and Applications", Academic Press, New York, **1990**.
21. P. M. Fanelle and A. K. Jain, "Recursive block coding: A new approach to transform coding", *IEEE Trans. Comm.*, **1986**, *C-34,* 161-179.
22. M. Bosi and G. Davidson, "High quality low bit-rate audio transform coding for transmission and multimedia applications", *J. Audio Eng. Soc.*, **1992**, *32,* 43-50.
23. F. J. MacWilliams and N. J. A. Sloane, "The Theory of Error Correcting Codes", Elsevier, Amsterdam, **1977**.
24. Y. T. Chan, "Wavelet Basics", Kluwer Academic Publishers, Norwell, MA, **1995**.
25. H. S. Malavar, "Signal Processing with Lapped Transforms", Artech House, Norwoed, MA, **1992**.

Maejo Int. J. Sci. Technol. **2010**, *4*(02), 235-249

26. S. G. Mallat, "A theory of multiresolution signal decomposition: The wavelet representation", *IEEE Trans. Pattern Anal. Mach. Intell.*, **1989**, *11*, 674-693.
27. G. Strang and T. Nguyer, "Wavelet and Filter Banks", Welesley-Cambridge Press, Welesley, MA, **1996**.
28. M. Vellerli and J. Kovacevic, "Wavelet and Sub-band Coding", Prentice-Hall, Englewood Cliffs, NJ, **1995**.
29. S. P. Nanavati and P. K. Panigrahi, "Wavelets: Applications to image compression-I", *J. Resonance*, **2005**, *10*, 52-59.
30. J. M. Shapiro, "Embedded image coding using zero-tree of wavelet coefficients", *IEEE Trans. Signal Process.*, **1993**, *41*, 3445-3462.
31. A. Said and W. A. Pearlman, "A new fast and efficient image coding based on set partitioning in hierarchical tree", *IEEE Trans. Circuits Syst. Video Technol.*, **1996**, *6*, 243-250.
32. D. Taubman, "High performance scalable image compression with EBCOT", *IEEE Trans. Image Process.*, **2000**, *9*, 1158-1170.
33. R. Buccigrossi and E. P. Simoncelli, "EPWIC: embedded predictive wavelet image coder", Technical Report #414 (May **1997**), GRASP Laboratory, University of Pennsylvania.
34. Z. Xiong, K. Ramachandran and M. T. Orchard, "Space frequency quantization for wavelet image coding", *IEEE Trans. Image Process.*, **1997**, *6*, 677-693.
35. M. Boliek, M. J. Gormish, E. L. Schwardz and A. Keith, "Next generation image compression and manipulations using CREW", Proceedings of International Conference on Image Processing, **1997**, Santa Barbara, CA, USA, pp. 567-570.
36. M. J. Tosi, J. D. Villasenor and F. Chen, "Stack-run image coding", *IEEE Trans. Circuits Syst. Video Technol.*, **1996,** *6*, 519-521.
37. J. Forment and S. Mallat, "Second Generation Compact Image Coding with Wavelets in Wavelets: A Tutorial in Theory and Applications", Vol. 2, Academic Press, New York, **1992**.
38. R. R. Coifman and M. V. Wickerhaurer, "Entropy based algorithm for best basis selection", *IEEE Trans. Inf. Theory*, **1992,** *38*, 713-718.
39. A. Gersh and R. M. Gray, "Vector Quantization and Signal Compression", Kluwer Academic Publishers, Norwell, MA, **1991**.
40. R. C. Colderbark, I. Daubechies, W. Sweldens and B. L. Yeo, "Wavelet transforms that map integer to integer", *Appl. Comput. Harmonic Anal.*, **1998,** *5*, 332-369.
41. N. Weiner, "Hermitian polynomials and Fourier analysis", *J. Math. Phys.*, **1929,** *8,* 70-73.
42. V. Namias, "The fractional order Fourier transform and it's application to quantum mechanics", *J. Inst. Math. Appl.,* **1980,** *25,* 241-265.
43. L. B. Almeida, "The Fractioal Fourier transform and time-frequency representation", *IEEE Trans. Signal Process.*, **1994,** *42,* 3084-3091.
44. O. N. Gerek and M. F. Erden, "Discrete fractional cosine transform", Proceedings of IEEE Conference on Signal Processing, Communications, Circuits, and Systems, **2000,** Istanbul, Turkey, pp. 322-329.
45. H. M. Ozaktas, O. Arikan, M. A. Kutay and G. Bozdagi. "Digital computation of the fractional Fourier transform", *IEEE Trans. Signal Process.*, **1996**, *44*, 2141-2150.

46. K. Singh, "Performance of discrete fractional Fourier transform classes in signal processing applications", *PhD. Thesis,* **2005**, Thapar Institute of Engineering and Technology, India**.**
47. W. Sweldens, "The lifting scheme: A custom-design construction of biorthogonal wavelets", Technical Report #7 (**1994**), Industrial Mathematics Initiative, Department of Mathematics, University of South Carolina.
48. W. Ding, F. Wu and S. Li, "Lifting-based wavelet transform with directionally spatial prediction", Proceedings of Picture Coding Symposium, **2004,** San Francisco, USA, pp. 483-488.
49. W. Ding, F. Wu, X. Wu, S. Li and H. Li, "Adaptive directional lifting based wavelet transform for image coding", *IEEE Trans. Image Process.*, **2007,** *16*, 416-427.
50. B. Zeng and J. Fu, "Directional discrete cosine transforms for image coding", Proceedings of IEEE Conference on Multimedia and Expo, **2006,** Toronto, Canada, pp. 721-724.
51. X. Hao , X. Jizheng and W. Feng, "Lifting based directional DCT-like transform for image coding", *IEEE Trans. Circuits Syst. Video Technol.*, **2007,** *17*, 1325-1335.
52. N. Ahmed, T. Natarajan and K. R. Rao, "Discrete cosine transform", *IEEE Trans. Comput.* **1974***,* *C-23,* 90-93.
53. G. B. Anderson and T. S. Huang, "Picture bandwidth compression by piecewise Fourier transformation", *IEEE Trans. Comm. Technol.*, **1971,** *CT-19,* 133-140.
54. W. Chen and W. K. Pratt, "Color image coding with slant transform", Proceedings of Symposium on Applications of Walsh Functions, **1973**, Washington, DC, USA, pp. 155-161.
55. W. K. Pratt, L. R. Welch and W. Chen, "Slant transform for image coding", *IEEE Trans. Comm.*, **1974,** *C-22,* 1075-1093.
56. A. Habibi and P. A. Wintz, "Image coding by linear transformation and block quantization", *IEEE Trans. Comm. Tech.*, **1971,** *CT-19,* 50-62.
57. W. K. Pratt, J. Kane and H. C. Andrews, "Hadamard transform image coding", *IEEE Proc. Comm.*, **1969,** *57*, 58-68.
58. P. A. Wintz, "Transform picture coding", *Proc. IEEE*, **1972,** *60*, 809-820.
59. W. H. Chen and C. H. Smith, "Adaptive coding of monochrome and color images", *IEEE Trans. Comm.,* **1977,** C-*25,* 1285-1292.
60. H. C. Andrews and C. L. Paterson, "Singular value decomposition (SVD) image coding", *IEEE Trans. Comm.*, **1976,** *24*, 425-432.
61. G. H. Golub and C. Reinsels, "Singular value decomposition and least square solutions", *Numer. Math.*, **1970,** *14*, 403-420.
62. C. S. McGoldrick, W. J. Doweling and A. Bury, "Image coding using the singular value decomposition and vector quantization", Proceedings of International Conference on Image Processing and Its Applications, **1995**, Edinburgh, Scotland, pp.296-300.
63. J. F. Yang and C. L. Lu, "Combined techniques of singular value decomposition and vector quantization", *IEEE Trans. Image Process.*, **1995,** *4*, 1141-1146.
64. W. Chen, C. H. Smith and S. Fralick, "A fast computational algorithm for the discrete cosine transform", *IEEE Trans. Comm.*, **1977**, *C-25,* 1004-1009.
65. P. Waldemar, "Image compression using singular value decomposition", *PhD. Thesis*, **1997,** Norwegian University of Science and Technology, Norway.

Maejo Int. J. Sci. Technol. **2010**, *4*(02), 235-249

66. P. Waldemar and P. A. Ramstad, "Hybrid KLT-SVD image compression", Proceedings of International Conference on Acoustics, Speech and Signal Processing, Vol.4, **1997,** Munich, Germany, pp. 2713-2716.
67. J. Chen, "Image compression with SVD", Lecture Notes (**2000),** University of California Davis, USA.
68. B. Arnold and A. McInnes, "An investigation into using singular value decomposition as a method of image compression", Technical Report (**2000**), College of Redwood, University of Canterbury, New Zealand.
69. A. Dapena and S. Ahalt, "A hybrid DCT-SVD image-coding algorithm", *IEEE Trans. Circuits Syst. Video Technol.*, **2002,** *12,* 114-121.
70. Y. Wongsawat, H. Ochoa, K. R. Rao and S. Oraintara, "A modified hybrid DCT-SVD image-coding system for color image", Proceedings of International Symposium on Communication and Information Technology, **2004,** Sapporo, Japan, pp. 766-769.
71. H. Ochoa and K. R. Rao, "A hybrid DWT-SVD image coding system", Proceedings of IEEE Symposium on Micro-Nano Mechatronics and Human Science, Vol. 2, **2003,** Cairo, Egypt, pp. 532-535.
72. H. Ochoa and K. R. Rao, "A new modified version of the HDWTSVD coding system for monochromatic images", *WSEAS Trans. Syst.*, **2006,** *5*, 1190-1195.
73. H. Ochoa and K. R. Rao, "A hybrid DWT-SVD image coding system (HDWTSVD) for colour images", *Int. J. System. Cybernet. Informat.*, **2005,** *1*, 64-69.
74. H. Ochoa and K. R. Rao, "A new modified hybrid DWT-SVD coding system for color images", *WSEAS Trans. Circuits Syst.*, **2005**, *4*, 1246-1253.
75. A. Ranade, S. S. Mahabalarao and S. Kale, "A variation on SVD based image compression", *J. Image Vision Comput.*, **2007**, *25*, 771-777.
76. S. O. Aase, J. H. Husoy and P. Waldemar, "A critique of SVD-based image coding systems," Proceedings of the 1999 IEEE International Symposium on Circuits and Systems on VLSI, Vol. 4, **1999**, Orlando, Florida, USA, pp.13-16.

© 2010 by Maejo University, San Sai, Chiang Mai, 50290 Thailand. Reproduction is permitted for noncommercial purposes.