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Full Paper

A novel method for estimating the parameter of a Gaussian AR(1) process with additive outliers

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Abstract: A novel estimator for a Gaussian first-order autoregressive [AR(1)] process with additive outliers is presented. A recursive median adjustment based on an α-trimmed mean was applied to the weighted symmetric estimator. The following estimators were considered: the weighted symmetric estimator ($\hat{\rho}_W$), the recursive-mean-adjusted weighted symmetric estimator ($\hat{\rho}_{R-W}$), the recursive-median-adjusted weighted symmetric estimator ($\hat{\rho}_{Rmd-W}$), and the weighted symmetric estimator using adjusted recursive median based on the α-trimmed mean ($\hat{\rho}_{Tm-Rmd-W}$). Using Monte Carlo simulations, the mean square errors (MSE) of the estimators were compared. Simulation results showed that the proposed estimator, $\hat{\rho}_{Tm-Rmd-W}$, provided a smaller MSE than those from $\hat{\rho}_W$, $\hat{\rho}_{R-W}$ and $\hat{\rho}_{Rmd-W}$ for almost all situations.

Keywords: parameter estimation, AR(1) process, recursive median, trimmed mean, additive outliers

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INTRODUCTION

In time series analysis, outliers or aberrant observations can have adverse impact on model identification, parameter estimation as well as forecasting. Outliers may occur because of human error in such activity as typing, recording and measuring mistakes or because of abrupt, short-term changes in the underlying process [1]. Fox [2], Abraham and Box [3] and Martin [4] discussed two kinds of outliers that can be found in time series data, namely additive outliers (AO) and innovational outliers (IO). An additive outlier corresponds to the situation in which a gross error of observation or recording error affects a single observation [2]. An innovational outlier affects not only the particular observation, but also subsequent observations [2]. In this study, the additive outliers are focused on because they are more harmful than innovational outliers [5]. A time series that does not contain any outliers is called an outlier-free series.

Suppose an outlier-free time series $\{X_t\,; t = 2,3,...,n\}$ follows a Gaussian first-order autoregressive process, AR(1), satisfying

$$X_t = \mu + \rho(X_{t-1} - \mu) + e_t, \tag{1}$$

where μ is the mean of the process, ρ is an autoregressive parameter, and $|\rho| < 1$ and e_t are independent and identically distributed random variables having normal distribution with zero mean and variance σ_e^2, i.e. $e_t \sim N(0,\sigma_e^2)$. For $\rho = 1$, the model (1) is called the random walk model; otherwise it is called a stationary AR(1) process when $|\rho| < 1$. The model (1) will be called a random walk model if $|\rho| = 1$; otherwise it is called a stationary model. In the case of ρ being close to one or near a non-stationary model, the mean and variance of this model change over time. Let the observed time series be denoted by $\{Y_t\}$. An additive outlier model is defined as

$$Y_t = X_t + \delta I_t^{(T)}, \tag{2}$$

where δ is the magnitude of the additive outliers and $I_t^{(T)}$ is an indicator variable such that $I_t^{(T)} = 1$ if $t = T$, and $I_t^{(T)} = 0$ if $t \neq T$. The model (2) can be interpreted that $\{X_t\}$ has additive outliers at time point T $(1 < T < n)$.

One of the well-known estimators of ρ is the ordinary least square (OLS) estimator. Although the OLS estimator has asymptotic normality for $|\rho| < 1$ [6-7], it has long been known that the OLS estimator can have a large bias [8-10]. In addition, Conover [11] indicated that the OLS estimator is sensitive to outliers. Therefore, useful improvements in the parameter estimation have been proposed so as to reduce the bias of the OLS estimator. Park and Fuller [12] proposed the weighted symmetric estimator (W) of ρ. A robust estimator for an autoregressive model was presented by Denby and Martin [13]. Guo [14] developed a simple and robust estimator for an AR(1) model. So and Shin [15] applied a recursive mean adjustment to the OLS estimator (ROLS) and they found that the mean square error of the ROLS estimator is smaller than the OLS estimator for $\rho \in (0,1)$. They also showed that the ROLS estimator has a coverage probability which is close to the nominal value. Niwitpong [16] applied the recursive mean adjustment to the weighted symmetric estimator (R-W) of Park and Fuller [12]. Panichkitkosolkul [17] proposed an estimator for an unknown mean Gaussian AR(1) model with additive outliers by applying the recursive median adjustment to the weighted symmetric estimator (Rmd-W). He found that the Rmd-W estimator is more efficient than the W or R-W estimator in terms of the mean square error for almost all situations.

In this paper, a new recursive median adjustment based on an α-trimmed mean [18] is applied to the weighted symmetric estimator (abbreviated Tm-Rmd-W) for model (1) when there are additive outliers in time series data. Because the outliers do not affect the trimmed mean and median values, the recursive mean adjustment is replaced by the new recursive median adjustment based on the α-trimmed mean. The aim of this paper is to compare four estimators, i.e. the weighted symmetric estimator $(\hat{\rho}_W)$, the weighted symmetric estimator based on the recursive mean adjustment $(\hat{\rho}_{R-W})$, the weighted symmetric estimator based on the recursive median adjustment $(\hat{\rho}_{Rmd-W})$, and the weighted symmetric estimator based on the recursive median adjustment by using the trimmed mean $(\hat{\rho}_{Tm-Rmd-W})$, in terms of mean square error (MSE) of the estimators.

METHODOLOGY

Park and Fuller [12] proposed the weighted symmetric estimator of ρ given by

$$\hat{\rho}_W = \frac{\sum_{t=2}^{n}(Y_t-\bar{Y})(Y_{t-1}-\bar{Y})}{\sum_{t=3}^{n}(Y_{t-1}-\bar{Y})^2+n^{-1}\sum_{t=1}^{n}(Y_t-\bar{Y})^2}. \qquad (3)$$

Niwitpong [16] replaces $\bar{Y}$ by $\bar{Y}_t=\frac{1}{t}\sum_{i=1}^{t}Y_i$ in (3). The estimator of ρ obtained as a result of this recursive mean adjustment is

$$\hat{\rho}_{R-W} = \frac{\sum_{t=2}^{n}(Y_t-\bar{Y}_t)(Y_{t-1}-\bar{Y}_{t-1})}{\sum_{t=3}^{n}(Y_{t-1}-\bar{Y}_{t-1})^2+n^{-1}\sum_{t=1}^{n}(Y_t-\bar{Y}_t)^2}. \qquad (4)$$

When there are outliers in time series data, it affects the recursive mean $\bar{Y}_t$ in (4). Panichkitkosolkul [17] replaces the recursive mean in (4) by the recursive median, $\tilde{Y}_t$. The estimator of ρ obtained as a result of the recursive median adjustment is

$$\hat{\rho}_{Rmd-W} = \frac{\sum_{t=2}^{n}(Y_t-\tilde{Y}_t)(Y_{t-1}-\tilde{Y}_{t-1})}{\sum_{t=3}^{n}(Y_{t-1}-\tilde{Y}_{t-1})^2+n^{-1}\sum_{t=1}^{n}(Y_t-\tilde{Y}_t)^2}, \qquad (5)$$

where $\tilde{Y}_t = median(Y_1, Y_2, ..., Y_t)$.

The effect of additive outliers on an estimator of ρ in model (1) can be reduced by using new recursive median adjustment based on an α-trimmed mean. The proposed recursive median values adjusted by an α-trimmed mean are derived from computing the α-trimmed mean of the recursive median. Therefore, the recursive median in (5) is replaced by a new recursive median. A novel estimator of ρ obtained as a result of this new recursive median adjustment is given by

$$\hat{\rho}_{Tm-Rmd-W} = \frac{\sum_{t=2}^{n}(Y_t-\bar{\tilde{Y}}_t)(Y_{t-1}-\bar{\tilde{Y}}_{t-1})}{\sum_{t=3}^{n}(Y_{t-1}-\bar{\tilde{Y}}_{t-1})^2+n^{-1}\sum_{t=1}^{n}(Y_t-\bar{\tilde{Y}}_t)^2}, \qquad (6)$$

where $\bar{\tilde{Y}}_t = \frac{1}{t-2[t\alpha]}\sum_{i=[t\alpha]+1}^{t-[t\alpha]}\tilde{Y}_{(i)}$; $\tilde{Y}_{(i)}$ denotes the ordered values of the recursive median $\tilde{Y}_i$, i.e. $\tilde{Y}_{(1)} \le \tilde{Y}_{(2)} \le \cdots \le \tilde{Y}_{(t)}$; α denotes the proportion of observations removed from both the upper and lower bounds, $0<\alpha<0.5$; and $[t\alpha]$ denotes the greatest integer not greater than $t\alpha$.

The performance of the proposed estimator for a Gaussian AR(1) process with additive outliers was examined via Monte Carlo simulations with particular emphasis on comparison between the novel and existing approaches. Data were generated from a Gaussian AR(1) process with additive outliers.

[$Y_1 \sim N(0, \frac{\sigma_e^2}{1-\rho^2})$ was generated and the time series of length $n+50$ was simulated but the time series used in calculations were $\{Y_{51}, Y_{52}, \ldots, Y_{n+50}\}$.] The following parameter values were used: (μ, σ_e)= (0, 1); ρ= 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8 and 0.9; sample sizes, n= 25, 50, 100 and 250; magnitude of AO effect, $\delta = 3\sigma_e$ and $5\sigma_e$; percentage of additive outliers, $p = 5\%$ and 10%; and fraction of data to be trimmed, $\alpha = 0.05$. All simulations were performed using programs written in R statistical software [19-21] with the number of simulation runs, $M = 10{,}000$. In addition, the additive outliers occurred randomly. The Monte Carlo simulation results of estimating MSE of these estimators, $\hat{\rho}_W$, $\hat{\rho}_{R-W}$, $\hat{\rho}_{Rmd-W}$ and $\hat{\rho}_{Tm-Rmd-W}$, are presented in the next section.

RESULTS

The simulation results are shown in Tables 1-2. The estimated MSE of all estimators, $\hat{\rho}_W$, $\hat{\rho}_{R-W}$, $\hat{\rho}_{Rmd-W}$ and $\hat{\rho}_{Tm-Rmd-W}$, is given by

$$MSE = Variance + Bias^2 = \frac{\sum_{j=1}^{M}(\hat{\rho}_j - \bar{\hat{\rho}})^2}{M-1} + \left[E(\hat{\rho}) - \rho\right]^2, \tag{7}$$

where $E(\hat{\rho}) = \bar{\hat{\rho}} = M^{-1}\sum_{j=1}^{M}\hat{\rho}_j$. As can be seen from Tables 1-2, the MSE of $\hat{\rho}_W$ is larger than those of the other estimators, especially when ρ is close to one and for small sample sizes. These values of MSE decrease as sample size gets larger (0.0412−0.1163 for $n = 25$; 0.0217−0.0507 for $n = 50$; 0.0114−0.0359 for $n = 100$; and 0.0050−0.0228 for $n = 250$). The $\hat{\rho}_W$ performs well for $n \geq 50$. On the other hand, the novel estimator, $\hat{\rho}_{Tm-Rmd-W}$, provides the lowest MSE in all scenarios that were considered except when the parameter ρ is small ($\rho = 0.1$ or 0.2). Additionally, $\hat{\rho}_{Tm-Rmd-W}$ performs very well in relation to the other three estimators. The proposed estimator, $\hat{\rho}_{Tm-Rmd-W}$ in (6), dominates all estimators since its MSE is the lowest for almost all cases. One reason for this is that the additive outliers do not affect the median and α-trimmed mean values. Moreover, the adjusted recursive median values applied in the formula of $\hat{\rho}_{Tm-Rmd-W}$ in (6) can also reduce the MSE of the estimator. For other estimators, the MSE of $\hat{\rho}_{Rmd-W}$ is less than that of $\hat{\rho}_{R-W}$ and $\hat{\rho}_W$ for almost all situations. The $\hat{\rho}_{Rmd-W}$ often ranks the second best after the proposed estimator. Furthermore, the values of MSE shown in Table 1 are less than those reported in Table 2 because the time series data in Table 1 have fewer outliers.

Using Monte Carlo simulations, the densities of estimates of the estimators, $\hat{\rho}_W$, $\hat{\rho}_{R-W}$, $\hat{\rho}_{Rmd-W}$ and $\hat{\rho}_{Tm-Rmd-W}$, are plotted in Figures 1-3 for each of $\rho = 0.2, 0.5, 0.9$, $n = 100$, $p = 10\%$ and $\delta = 3\sigma_e$. As can be seen from these figures, the estimated densities of all estimators are not different and they are symmetric when $\rho = 0.2$ and 0.5. However, when ρ is equal to 0.9, the density estimates of all estimators are skewed to the left. In addition, the difference between the mode of the estimated density of $\hat{\rho}_{Tm-Rmd-W}$ and true parameter ρ is smallest compared to those obtained with other estimators.

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Table 1. Estimated values of MSE of $\hat{\rho}_W$, $\hat{\rho}_{R-W}$, $\hat{\rho}_{Rmd-W}$ and $\hat{\rho}_{Tm-Rmd-W}$ when percentage of additive outliers, $p = 5\%$

n	ρ	$\delta = 3\sigma_e$				$\delta = 5\sigma_e$			
		W	R-W	Rmd-W	Tm-Rmd-W	W	R-W	Rmd-W	Tm-Rmd-W
25	0.1	0.0412	0.0367	0.0346	0.0428	0.0394	0.0347	0.0318	0.0339
	0.2	0.0487	0.0417	0.0389	0.0407	0.0539	0.0456	0.0420	0.0357
	0.3	0.0597	0.0500	0.0471	0.0412	0.0767	0.0647	0.0609	0.0461
	0.4	0.0714	0.0594	0.0562	0.0418	0.1040	0.0887	0.0849	0.0600
	0.5	0.0825	0.0693	0.0664	0.0453	0.1309	0.1129	0.1102	0.0756
	0.6	0.0956	0.0810	0.0791	0.0509	0.1640	0.1433	0.1422	0.0964
	0.7	0.1053	0.0900	0.0893	0.0550	0.1933	0.1705	0.1690	0.1125
	0.8	0.1106	0.0963	0.0963	0.0581	0.2093	0.1861	0.1851	0.1216
	0.9	0.1163	0.1038	0.1038	0.0626	0.2221	0.1994	0.2002	0.1308
50	0.1	0.0217	0.0200	0.0191	0.0236	0.0220	0.0202	0.0189	0.0197
	0.2	0.0252	0.0226	0.0209	0.0214	0.0335	0.0300	0.0276	0.0229
	0.3	0.0310	0.0276	0.0259	0.0223	0.0481	0.0431	0.0400	0.0296
	0.4	0.0377	0.0334	0.0312	0.0232	0.0672	0.0608	0.0575	0.0399
	0.5	0.0443	0.0395	0.0377	0.0255	0.0881	0.0804	0.0777	0.0532
	0.6	0.0491	0.0442	0.0430	0.0270	0.1044	0.0963	0.0936	0.0628
	0.7	0.0507	0.0460	0.0450	0.0270	0.1176	0.1087	0.1068	0.0701
	0.8	0.0493	0.0453	0.0447	0.0255	0.1177	0.1100	0.1084	0.0691
	0.9	0.0440	0.0415	0.0412	0.0233	0.1058	0.0998	0.0988	0.0610
100	0.1	0.0114	0.0107	0.0101	0.0116	0.0139	0.0130	0.0114	0.0111
	0.2	0.0152	0.0140	0.0128	0.0116	0.0247	0.0230	0.0197	0.0157
	0.3	0.0206	0.0190	0.0175	0.0133	0.0403	0.0378	0.0337	0.0252
	0.4	0.0262	0.0243	0.0226	0.0155	0.0585	0.0553	0.0511	0.0380
	0.5	0.0315	0.0294	0.0277	0.0180	0.0788	0.0750	0.0707	0.0522
	0.6	0.0359	0.0337	0.0323	0.0204	0.0948	0.0908	0.0867	0.0632
	0.7	0.0358	0.0338	0.0325	0.0197	0.1039	0.0998	0.0964	0.0680
	0.8	0.0318	0.0303	0.0293	0.0172	0.0977	0.0940	0.0915	0.0620
	0.9	0.0229	0.0222	0.0217	0.0121	0.0719	0.0695	0.0677	0.0427
250	0.1	0.0050	0.0048	0.0045	0.0046	0.0072	0.0069	0.0055	0.0050
	0.2	0.0080	0.0076	0.0068	0.0054	0.0160	0.0154	0.0127	0.0102
	0.3	0.0119	0.0113	0.0103	0.0074	0.0298	0.0289	0.0253	0.0203
	0.4	0.0164	0.0157	0.0146	0.0102	0.0462	0.0450	0.0410	0.0333
	0.5	0.0201	0.0194	0.0183	0.0126	0.0624	0.0610	0.0570	0.0462
	0.6	0.0228	0.0221	0.0212	0.0144	0.0752	0.0737	0.0701	0.0561
	0.7	0.0224	0.0218	0.0210	0.0138	0.0792	0.0777	0.0750	0.0585
	0.8	0.0176	0.0172	0.0167	0.0105	0.0685	0.0673	0.0654	0.0481
	0.9	0.0101	0.0100	0.0097	0.0056	0.0409	0.0403	0.0391	0.0261

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Table 2. Estimated values of MSE of $\hat{\rho}_W$, $\hat{\rho}_{R-W}$, $\hat{\rho}_{Rmd-W}$ and $\hat{\rho}_{Tm-Rmd-W}$ when percentage of additive outliers, $p = 10\%$

n	ρ	$\delta = 3\sigma_e$				$\delta = 5\sigma_e$			
		W	R-W	Rmd-W	Tm-Rmd-W	W	R-W	Rmd-W	Tm-Rmd-W
25	0.1	0.0443	0.0395	0.0363	0.0413	0.0439	0.0388	0.0335	0.0343
	0.2	0.0570	0.0484	0.0440	0.0411	0.0663	0.0571	0.0493	0.0421
	0.3	0.0735	0.0616	0.0560	0.0443	0.0995	0.0862	0.0765	0.0612
	0.4	0.0927	0.0783	0.0730	0.0532	0.1410	0.1237	0.1125	0.0859
	0.5	0.1159	0.0991	0.0936	0.0639	0.1869	0.1654	0.1546	0.1165
	0.6	0.1379	0.1193	0.1141	0.0759	0.2291	0.2045	0.1924	0.1430
	0.7	0.1529	0.1334	0.1298	0.0841	0.2821	0.2542	0.2437	0.1781
	0.8	0.1700	0.1496	0.1467	0.0941	0.3176	0.2877	0.2776	0.2010
	0.9	0.1754	0.1568	0.1550	0.0987	0.3370	0.3059	0.2971	0.2101
50	0.1	0.0241	0.0222	0.0206	0.0229	0.0272	0.0250	0.0202	0.0208
	0.2	0.0337	0.0301	0.0266	0.0234	0.0450	0.0409	0.0317	0.0274
	0.3	0.0469	0.0419	0.0373	0.0281	0.0749	0.0687	0.0557	0.0450
	0.4	0.0622	0.0560	0.0504	0.0355	0.1093	0.1013	0.0860	0.0677
	0.5	0.0804	0.0731	0.0677	0.0461	0.1515	0.1417	0.1259	0.0987
	0.6	0.0936	0.0858	0.0808	0.0536	0.1888	0.1773	0.1611	0.1238
	0.7	0.1032	0.0954	0.0910	0.0594	0.2228	0.2102	0.1955	0.1465
	0.8	0.1035	0.0962	0.0929	0.0591	0.2365	0.2233	0.2119	0.1529
	0.9	0.0912	0.0859	0.0836	0.0511	0.2228	0.2110	0.2022	0.1385
100	0.1	0.0132	0.0123	0.0109	0.0118	0.0161	0.0151	0.0110	0.0114
	0.2	0.0206	0.0190	0.0162	0.0134	0.0324	0.0304	0.0218	0.0185
	0.3	0.0312	0.0290	0.0253	0.0185	0.0581	0.0551	0.0425	0.0350
	0.4	0.0434	0.0407	0.0364	0.0256	0.0890	0.0852	0.0702	0.0574
	0.5	0.0563	0.0531	0.0488	0.0337	0.1243	0.1197	0.1041	0.0849
	0.6	0.0674	0.0640	0.0602	0.0411	0.1576	0.1522	0.1371	0.1103
	0.7	0.0700	0.0668	0.0637	0.0425	0.1804	0.1746	0.1620	0.1270
	0.8	0.0652	0.0625	0.0602	0.0383	0.1821	0.1762	0.1664	0.1239
	0.9	0.0469	0.0454	0.0439	0.0262	0.1437	0.1393	0.1333	0.0912
250	0.1	0.0064	0.0061	0.0049	0.0048	0.0093	0.0090	0.0052	0.0052
	0.2	0.0126	0.0121	0.0097	0.0076	0.0244	0.0236	0.0152	0.0133
	0.3	0.0222	0.0214	0.0183	0.0139	0.0473	0.0462	0.0339	0.0295
	0.4	0.0328	0.0318	0.0283	0.0215	0.0772	0.0757	0.0610	0.0538
	0.5	0.0443	0.0431	0.0398	0.0306	0.1088	0.1070	0.0914	0.0802
	0.6	0.0518	0.0505	0.0477	0.0361	0.1381	0.1361	0.1213	0.1054
	0.7	0.0533	0.0521	0.0500	0.0369	0.1568	0.1546	0.1424	0.1208
	0.8	0.0452	0.0443	0.0427	0.0300	0.1500	0.1478	0.1402	0.1131
	0.9	0.0254	0.0251	0.0243	0.0154	0.1002	0.0987	0.0951	0.0695

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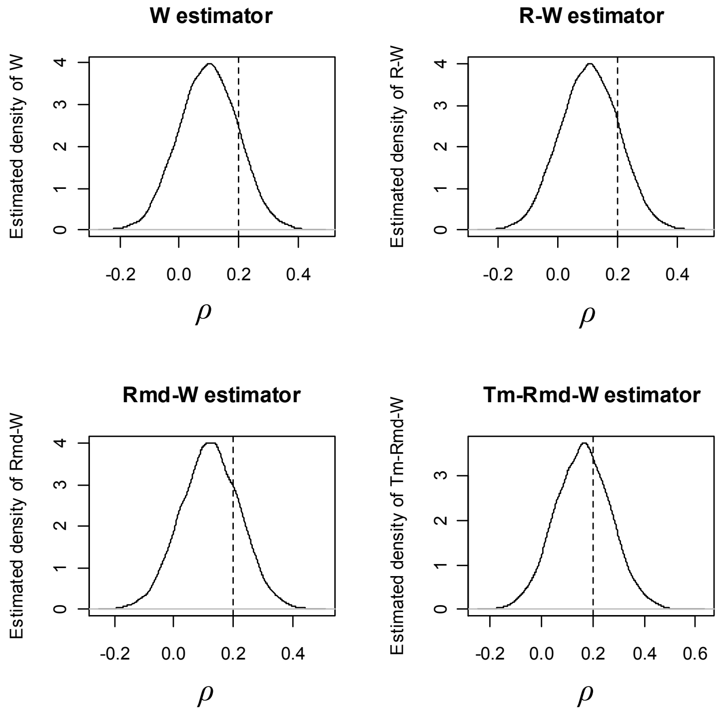

Figure 1. Comparison of all density estimates when $\rho = 0.2$, $n = 100$, $p = 10\%$ and $\delta = 3\sigma_e$

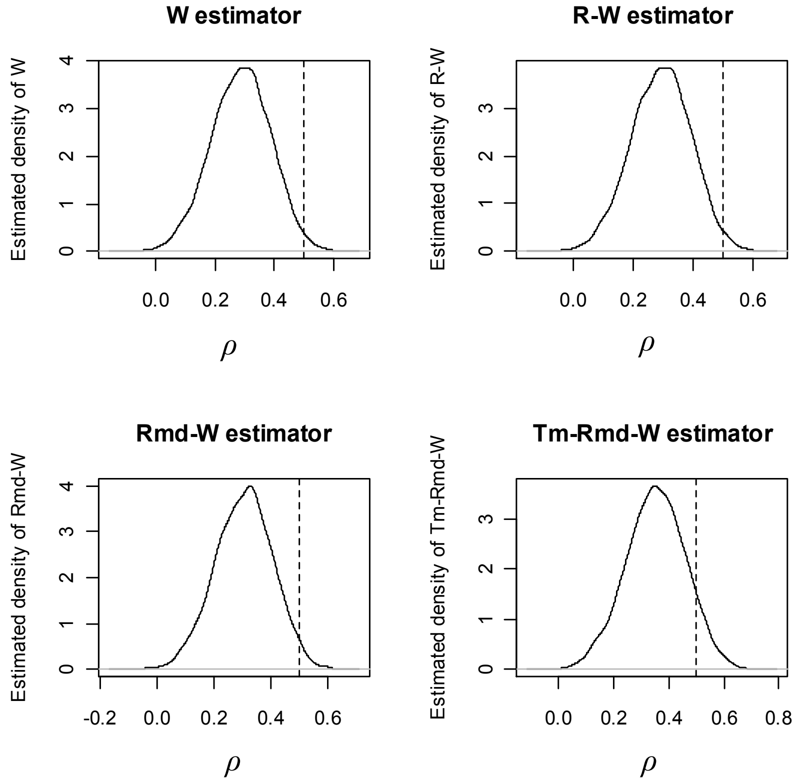

Figure 2. Comparison of all density estimates when $\rho = 0.5$, $n = 100$, $p = 10\%$ and $\delta = 3\sigma_e$

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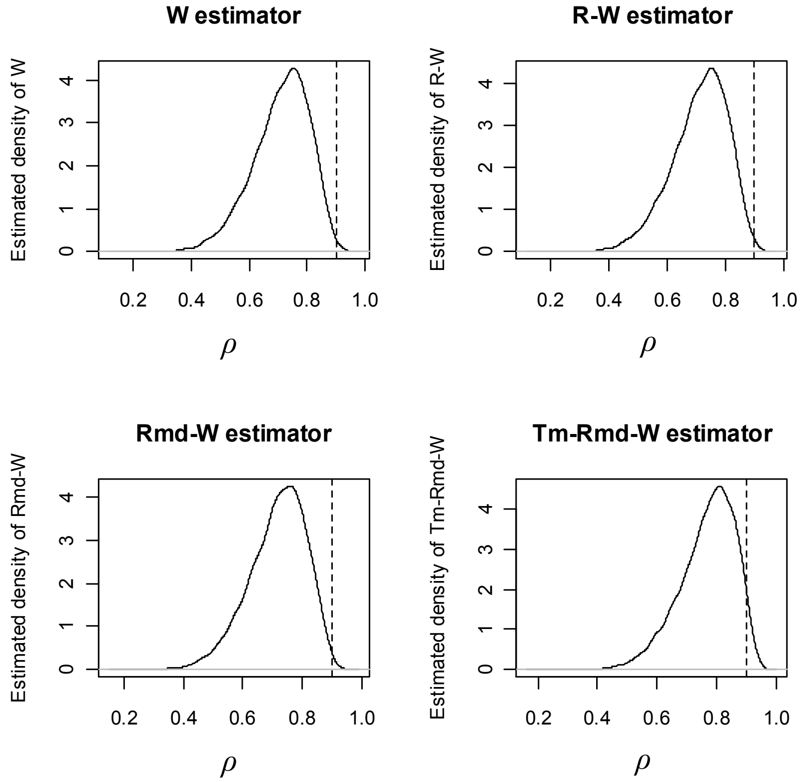

Figure 3. Comparison of all density estimates when $\rho = 0.9$, $n = 100$, $p = 10\%$ and $\delta = 3\sigma_e$

REAL DATA EXAMPLE

To illustrate the application of the estimators which have been proposed in the previous section, the yearly real exchange rates between USA and Sudan from 1970 to 2008 (base year: 2005) are used. A series giving a total of 39 observations was collected from the Economic Research Service, United States Department of Agriculture [22]. The time series plot, the sample autocorrelation function (ACF) and the sample partial autocorrelation function (PACF), as shown in Figures 4-5, suggest that an AR(1) model is suitable. The additive outliers of this series were detected by using an iterative detecting procedure proposed by Chang et al. [23] via the R statistical software [1]. It was found that the time indices of potential AO are $t = 22$ and 23 (year 1991 and 1992). All estimators, $\hat{\rho}_W$, $\hat{\rho}_{R-W}$, $\hat{\rho}_{Rmd-W}$ and $\hat{\rho}_{Tm-Rmd-W}$ and their standard errors and variances were also constructed (Table3). As presented in Table 4, the proposed estimator, $\hat{\rho}_{Tm-Rmd-W}$, provides about 11.6%, 11.1% and 10.0% less standard error than those of the $\hat{\rho}_W$, $\hat{\rho}_{R-W}$ and $\hat{\rho}_{Rmd-W}$ respectively, which confirms that the proposed estimator, $\hat{\rho}_{Tm-Rmd-W}$, is much better than the other estimators.

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Figure 4. The US/Sudan annual real exchange rates from 1970 to 2008

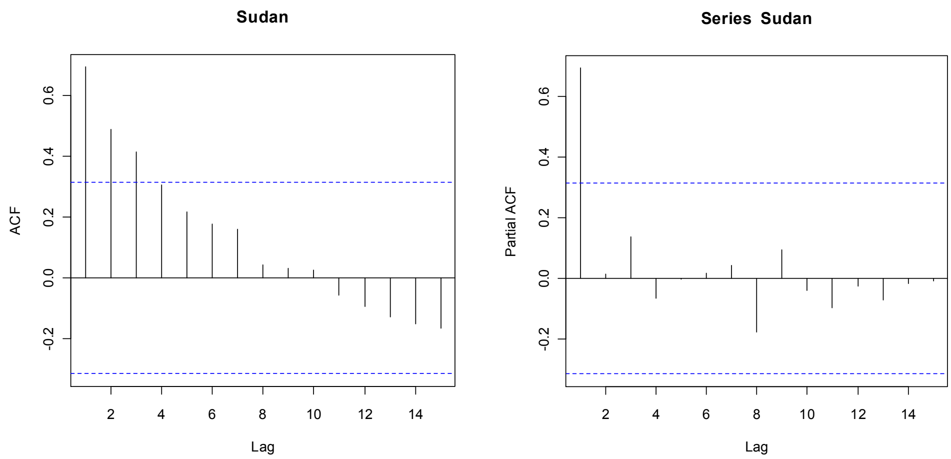

Figure 5. ACF and PACF of the US/Sudan real exchange rates

Table 3. The standard errors and variances of all estimators

$SE(\hat{\rho}_W) = \dfrac{\hat{\sigma}_W}{\sqrt{\sum_{t=2}^{n}(Y_{t-1}-\bar{Y})^2}}$,	$\hat{\sigma}_W^2 = \dfrac{\sum_{t=2}^{n}(Y_t-\bar{Y}-\hat{\rho}_W(Y_{t-1}-\bar{Y}))^2}{n-2}$
$SE(\hat{\rho}_{R-W}) = \dfrac{\hat{\sigma}_{R-W}}{\sqrt{\sum_{t=2}^{n}(Y_{t-1}-\bar{Y}_{t-1})^2}}$,	$\hat{\sigma}_{R-W}^2 = \dfrac{\sum_{t=2}^{n}(Y_t-\bar{Y}_t-\hat{\rho}_{R-W}(Y_{t-1}-\bar{Y}_{t-1}))^2}{n-2}$
$SE(\hat{\rho}_{Rmd-W}) = \dfrac{\hat{\sigma}_{Rmd-W}}{\sqrt{\sum_{t=2}^{n}(Y_{t-1}-\tilde{Y}_{t-1})^2}}$,	$\hat{\sigma}_{Rmd-W}^2 = \dfrac{\sum_{t=2}^{n}(Y_t-\tilde{Y}_t-\hat{\rho}_{Rmd-W}(Y_{t-1}-\tilde{Y}_{t-1}))^2}{n-2}$
$SE(\hat{\rho}_{Tm-Rmd-W}) = \dfrac{\hat{\sigma}_{Tm-Rmd-W}}{\sqrt{\sum_{t=2}^{n}(Y_{t-1}-\bar{\tilde{Y}}_{t-1})^2}}$,	$\hat{\sigma}_{Tm-Rmd-W}^2 = \dfrac{\sum_{t=2}^{n}(Y_t-\bar{\tilde{Y}}_t-\hat{\rho}_{Tm-Rmd-W}(Y_{t-1}-\bar{\tilde{Y}}_{t-1}))^2}{n-2}$

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Table 4. Parameter estimates and standard errors of estimators for US/Sudan real exchange rates series

Method	Estimate	Standard error (SE)
W	0.6766	0.11871
R-W	0.6799	0.11820
Rmd-W	0.6860	0.11706
Tm-Rmd-W	0.7435	0.10641

CONCLUSIONS

A novel estimator for a Gaussian AR(1) process with additive outliers has been proposed. This estimator of the autoregressive parameter is obtained by applying a recursive median adjustment based on an α-trimmed mean to the weighted symmetric estimator. The adjusted recursive median values are derived from computation of the α-trimmed mean of the recursive median. The weighted symmetric estimator ($\hat{\rho}_W$), the recursive-mean-adjusted weighted symmetric estimator ($\hat{\rho}_{R-W}$), the recursive-median-adjusted weighted symmetric estimator ($\hat{\rho}_{Rmd-W}$) and the novel estimator ($\hat{\rho}_{Tm-Rmd-W}$) are compared in this study. The result shows that the novel estimator gives the best performance in terms of the mean square error of the estimators for almost all scenarios.

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