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Communication

Determination of production-shipment policy using a two-phase algebraic approach

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Abstract: The optimal production-shipment policy for end products using mathematical modeling and a two-phase algebraic approach is investigated. A manufacturing system with a random defective rate, a rework process, and multiple deliveries is studied with the purpose of deriving the optimal replenishment lot size and shipment policy that minimises total production-delivery costs. The conventional method uses differential calculus on the system cost function to determine the economic lot size and optimal number of shipments for such an integrated vendor-buyer system, whereas the proposed two-phase algebraic approach is a straightforward method that enables practitioners who may not have sufficient knowledge of calculus to manage real-world systems more effectively.

Keywords: manufacturing system, replenishment lot size, delivery, two-phase algebraic approach, random defective rate

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INTRODUCTION

With the purpose of minimising total set-up and holding costs, inventory controllers in most companies need to address two basic issues for items they routinely stock: when to start replenishment and how much to refill. For items made in-house by manufacturing firms, production planners must, without exception, decide when to initiate a production run and how many items to produce in a run [1]. An inventory model that uses mathematical techniques to derive the most economical production lot was first proposed by Taft [2] several decades ago. This is also known as the economic production quantity (EPQ) model [3].

The classic EPQ model assumes a continuous inventory issuing policy to satisfy product demand. However, in real-world vendor-buyer systems, multiple or periodic deliveries of end items are commonly adopted. Hence, the determination of the optimal number of shipments for a finished lot becomes a critical issue to such a vendor-buyer system in terms of production-delivery cost minimisation. Schwarz [4] examined a one-warehouse N-retailer deterministic inventory system with the objective of determining the stock policy that minimises the long-run average system cost per unit time. He derived optimal solutions along with a few necessary properties for a one-retailer and N-identical-retailer problems. Heuristic solutions for the general problem were also suggested. Goyal [5] studied an integrated single-supplier-single-customer problem and presented a method that is typically applicable to those inventory problems where a product is procured by a single customer from a single supplier using examples to demonstrate the proposed model. Studies related to various aspects of supply chain optimisation have since been extensively carried out [e.g. 6-13]. The classic EPQ model also assumes that all items produced are of perfect quality. However, in a real-life manufacturing environment, the generation of nonconforming items is almost inevitable due to process deterioration or various other factors. In the past decades, many studies have attempted to address the issues of defective products and quality assurance in production systems [e.g. 14-24].

Shih [14] extended two inventory models to the case where the proportion of defective units in the accepted lot is a random variable with known probability distribution. Optimal solutions to the amended systems were developed and comparisons with the traditional models were also presented via numerical examples. Moinzadeh and Aggarwal [17] studied a production-inventory system that was subjected to random disruptions. They assumed that the time between breakdowns is exponential, the restoration times are constant, and excess demand is back-ordered. An (s, S) policy was proposed and the policy parameters that minimise the expected total cost per unit time were investigated. A procedure for finding the optimal values of the policy was also developed. Makis [18] investigated the optimal lot sizing and inspection policy for an economic manufacturing quantity (EMQ) model with imperfect inspections and assumed that the process could be monitored through inspections and that both the lot size and the inspection schedule were subjected to control. It was assumed that the in-control periods were generally distributed and the inspections imperfect. Using Lagrange's method and solving a non-linear equation, a two-dimensional search procedure was proposed for finding the optimal lot sizing and inspection policy. Rahim and Ben-Daya [19] studied the simultaneous effects of deteriorating product items and deteriorating production processes on the economic production quantity, inspection schedules, and economic design of control charts. Deterioration times for both product and process were assumed to follow an arbitrary distribution, and the product quality characteristic was assumed to be normally distributed. Numerical examples were provided to demonstrate the usage of their models. Chiu et al. [21] studied the optimal replenishment policy for the EMQ model with rework failure, backlogging and random breakdowns. Mathematical modelling and cost analysis were employed in their study, along with a renewal reward theorem for dealing with variable cycle length. They derived a long-run average cost function for their proposed model and proved that it was a convex function. Finally, they obtained an optimal replenishment policy for such an imperfect EMQ model.

Recently, an algebraic method of determining the economic order quantity (EOQ) model with backlogging was introduced by Grubbström and Erdem [25]. They used algebraic derivation to find the optimal order quantity without reference to the first- or second-order derivatives. Similar methodologies have been applied to solve various aspects of supply chain optimisation [26-28]. This paper extends such an approach in order to re-examine a manufacturing system with a random defective rate, a rework process, and multiple deliveries of its end product [13].

METHODS

We present a two-phase algebraic approach [13] in order to re-examine a manufacturing system with a random defective rate, a rework process, and multiple shipments of finished items. Such a specific model is described as follows. Assume a production system has an annual production rate P and randomly produces a proportion x of defective items during its uptime at a production rate d. All manufactured items are screened and the inspection cost is included in the unit production cost C. Non-conforming products fall into two groups: the scrap (a proportion of θ) and the repairable ($1-\theta$). The rework process has a rate of P_1 units per year and commences immediately after regular production in each cycle. A proportion θ_1 of reworked items fails during rework and is treated as scrap. Under regular supply, the constant production rate P must be larger than the sum of the demand rate λ and the production rate of defective items d, i.e. $(P-d-\lambda) > 0$, where d can be expressed as $d = Px$. Let d_1 denote the production rate of scrap during rework; d_1 can then be expressed as $d_1 = P_1\theta_1$. Furthermore, the proposed system considers a multi-delivery policy for the end items with quality assurance. That is, the finished items can only be delivered to the customers if the whole lot is quality assured at the end of the reworking process. A fixed quantity of n installments of the finished batch is delivered to customers at fixed interval of time during production downtime t_3 (Figure 1). Other notations used in the proposed system are listed below.

t_1 = regular production time in the proposed model,
t_2 = time required to rework defective items,
t_3 = time required to deliver all perfect-quality end products,
t_n = fixed interval of time between each installment of finished end products delivered during production downtime t_3,
T = cycle length,
Q = manufacturing batch size—the decision variable,
n = number of fixed-quantity installments of finished batch to be delivered to customers—the decision variable,
H_1 = maximum level of on-hand inventory when regular production ends,
H = maximum level of on-hand inventory when the rework process finishes,
$I(t)$ = on-hand inventory of perfect quality end items at manufacturer's end at time t,
$TC(Q,n)$ = total production-inventory-delivery costs per cycle,
K = set-up cost per cycle,
C = unit production cost,
h = unit holding cost,
C_R = unit rework cost,
h_1 = holding cost for each reworked item,

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C_S = disposal cost per scrap item,
K_1 = fixed delivery cost per shipment,
C_T = delivery cost per item shipped to customers,
φ = overall scrap rate per cycle (sum of scrap rates in periods t_1 and t_2),
h_2 = holding cost for each item kept by customer,
$E[TCU(Q,n)]$ = long-run average cost per unit time.

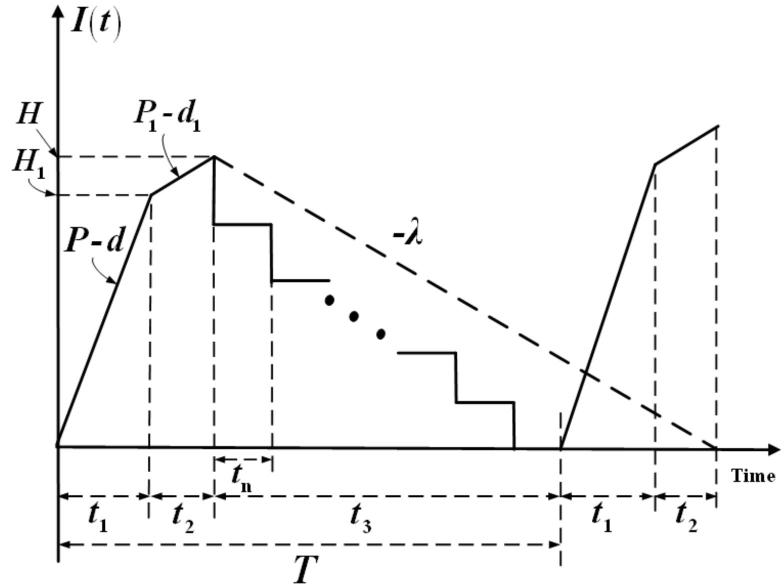

Figure 1. On-hand inventory of perfect end items in the proposed model with random defective rate, reworking and multi-delivery policy [13]

With reference to Figure 1, the total production-inventory-delivery cost per cycle, $TC(Q, n)$, consists of the following. (a) set-up cost and variable manufacturing costs per cycle; (b) total quality costs including variable repairing costs, holding costs for reworked items, and disposal costs for scrap items per cycle; (c) fixed and variable delivery costs per cycle; (d) total holding costs at the manufacturer's end for all items produced in the periods t_1, t_2 and t_3; and (e) total holding costs at the customer's end for all items stocked in t_3:

$$\begin{aligned} TC(Q,n) &= K + CQ + C_R\left[x(1-\theta)Q\right] + h_1 \cdot \frac{P_1 \cdot t_2}{2} \cdot (t_2) + C_S\left[x\varphi Q\right] + nK_1 \\ &+ C_T\left[Q(1-\varphi x)\right] + h\left[\frac{H_1 + dt_1}{2}(t_1) + \frac{H_1 + H}{2}(t_2) + \left(\frac{n-1}{2n}\right)Ht_3\right] \\ &+ \frac{h_2}{2}\left[\frac{H}{n}t_3 + T(H - \lambda t_3)\right] \end{aligned} \tag{1}$$

With further derivations, the long-run average cost per unit time $E[TCU(Q,n)]$ for the proposed system can be written as follows (see mathematical modelling section in Chiu et al. [13]):

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$$
\begin{aligned}
E\left[TCU(Q,n)\right] = \frac{E\left[TC(Q,n)\right]}{E[T]} &= \frac{C\lambda}{1-\varphi E[x]} + \frac{(K+nK_1)\lambda}{Q(1-\varphi E[x])} + C_{\mathrm{T}}\lambda \\
&+ \frac{C_{\mathrm{R}} E[x](1-\theta)\lambda}{(1-\varphi E[x])} + \frac{h_1 (E[x])^2 Q\lambda(1-\theta)^2}{2P_1(1-\varphi E[x])} + \frac{C_{\mathrm{S}} E[x]\varphi\lambda}{(1-\varphi E[x])} \\
&+ \frac{hQ\lambda}{2P(1-\varphi E[x])} + \frac{hQ\lambda}{2P_1(1-\varphi E[x])}\left[\left(2E[x]-(E[x])^2-\varphi(E[x])^2\right)(1-\theta)\right] \\
&+ \left(1-\frac{1}{n}\right)\left[\frac{hQ(1-\varphi E[x])}{2} - \frac{hQ\lambda}{2P} - \frac{hQE[x](1-\theta)\lambda}{2P_1}\right] \\
&+ \left(\frac{1}{n}\right)\frac{h_2 Q}{2}(1-\varphi E[x]) + \left(1-\frac{1}{n}\right)\frac{h_2 Q\lambda}{2P} + \frac{h_2 Q}{2}\left[\left(1-\frac{1}{n}\right)\frac{E[x]\lambda(1-\theta)}{P_1}\right]
\end{aligned} \tag{2}
$$

Derivation of Optimal Policy using Two-phase Algebraic Approach

Unlike the conventional method, which uses differential calculus on the cost function E[*TCU*(*Q, n*)] to find the optimal point [13], a straightforward two-phase algebraic approach to determining the optimal production-shipment policy for the proposed model is adopted here.

*Phase 1: Derivation of n**

It can be seen that Eq.2 has two decision variables, namely Q and n. Moreover, there are several different forms of these decision variables in the right-hand side of Eq.2, e.g., Q, Q^{-1}, nQ^{-1} and Qn^{-1}. Therefore, Eq.2 can be rearranged as

$$
\begin{aligned}
E\left[TCU(Q,n)\right] &= \frac{C\lambda}{1-\varphi E[x]} + C_{\mathrm{T}}\lambda + \frac{C_{\mathrm{R}} E[x](1-\theta)\lambda}{(1-\varphi E[x])} + \frac{C_{\mathrm{S}} E[x]\varphi\lambda}{(1-\varphi E[x])} \\
&+ \left\{\frac{\lambda}{(1-\varphi E[x])}\cdot\left[\frac{h_1(E[x])^2(1-\theta)^2}{2P_1} + \frac{h}{2P} + \frac{h}{2P_1}\left[\left(2E[x]-(E[x])^2-\varphi(E[x])^2\right)(1-\theta)\right]\right]\right\}Q \\
&+ \left[\frac{h(1-\varphi E[x])}{2} - (h-h_2)\left[\frac{\lambda}{2P} + \frac{E[x](1-\theta)\lambda}{2P_1}\right]\right]Q + \frac{K\lambda}{(1-\varphi E[x])}Q^{-1} \\
&+ \frac{K_1\lambda}{(1-\varphi E[x])}\left(nQ^{-1}\right) \\
&+ \left[(h-h_2)\left[-\frac{(1-\varphi E[x])}{2} + \frac{\lambda}{2P} + \frac{E[x](1-\theta)\lambda}{2P_1}\right]\right]\left(n^{-1}Q\right)
\end{aligned} \tag{3}
$$

or

$$E\left[TCU(Q,n)\right] = \alpha_1 + \alpha_2(Q) + \alpha_3\left(Q^{-1}\right) + \alpha_4\left(nQ^{-1}\right) + \alpha_5\left(n^{-1}Q\right) \tag{4}$$

where α_1, α_2, α_3, α_4 and α_5 denote:

$$\alpha_1 = \frac{C\lambda}{1-\varphi E[x]} + C_{\mathrm{T}}\lambda + \frac{C_{\mathrm{R}} E[x](1-\theta)\lambda}{(1-\varphi E[x])} + \frac{C_{\mathrm{S}} E[x]\varphi\lambda}{(1-\varphi E[x])} \tag{5}$$

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$$\alpha_2 = \frac{\lambda}{\left(1-\varphi E[x]\right)} \cdot \left[\frac{h_1 \left(E[x]\right)^2 (1-\theta)^2}{2P_1} + \frac{h}{2P} + \frac{h}{2P_1}\left[\left(2E[x] - \left(E[x]\right)^2 - \varphi\left(E[x]\right)^2\right)(1-\theta)\right] \right] + \left[\frac{h\left(1-\varphi E[x]\right)}{2} - (h-h_2)\left[\frac{\lambda}{2P} + \frac{E[x](1-\theta)\lambda}{2P_1}\right]\right] \tag{6}$$

$$\alpha_3 = \frac{K\lambda}{\left(1-\varphi E[x]\right)} \tag{7}$$

$$\alpha_4 = \frac{K_1\lambda}{\left(1-\varphi E[x]\right)} \tag{8}$$

$$\alpha_5 = (h-h_2)\left[-\frac{\left(1-\varphi E[x]\right)}{2} + \frac{\lambda}{2P} + \frac{E[x](1-\theta)\lambda}{2P_1}\right] \tag{9}$$

With further rearrangements, Eq.4 becomes

$$E\left[TCU(Q,n)\right] = \alpha_1 + Q^{-1}\left[\alpha_2 \cdot Q^2 + \alpha_3\right] + \left(n^{-1}Q\right)\left[\alpha_4\left(nQ^{-1}\right)^2 + \alpha_5\right] \tag{10}$$

$$E\left[TCU(Q,n)\right] = \alpha_1 + Q^{-1}\left[\left(\sqrt{\alpha_2}\cdot Q\right) - \sqrt{\alpha_3}\right]^2 + \left(n^{-1}Q\right)\left[\left(nQ^{-1}\sqrt{\alpha_4}\right) - \sqrt{\alpha_5}\right]^2 + 2\sqrt{\alpha_2\cdot\alpha_3} + 2\sqrt{\alpha_4\cdot\alpha_5} \tag{11}$$

Eq.11 will be minimised if its second and third terms in it equal zero. That is:

$$Q = \sqrt{\frac{\alpha_3}{\alpha_2}} \tag{12}$$

$$n = \sqrt{\frac{\alpha_5}{\alpha_4}} \cdot Q \tag{13}$$

Substituting Eq.6 and 7 into Eq.12, and then substituting Eq.8, 9 and 12 into Eq.13, the optimal number of shipments n^* is

$$n = \sqrt{\frac{\alpha_5\cdot\alpha_3}{\alpha_4\cdot\alpha_2}} = \sqrt{\frac{(h-h_2)\left[-\left(1-\varphi E[x]\right) + \frac{\lambda}{P} + \frac{E[x](1-\theta)\lambda}{P_1}\right]\cdot\frac{K}{K_1}}{\frac{\lambda}{\left(1-\varphi E[x]\right)}\cdot\left[\frac{h_1\left(E[x]\right)^2(1-\theta)^2}{P_1} + \frac{h}{P} + \frac{h}{P_1}\left[\left(2E[x] - \left(E[x]\right)^2 - \varphi\left(E[x]\right)^2\right)(1-\theta)\right]\right] + \left[h\left(1-\varphi E[x]\right) - (h-h_2)\left[\frac{\lambda}{P} + \frac{E[x](1-\theta)\lambda}{P_1}\right]\right]}} \tag{14}$$

It is noted that Eq.14 is identical to that obtained using the conventional differential calculus method [13]. We can also see that although in real-world situation the number of deliveries takes integer values only, Eq. 14 results in a real number. In order to locate the integer value of n^* that minimises the long-run average cost for the proposed system, the two adjacent integers to n must be examined respectively for cost minimisation [11]. Let n^+ denote the smallest integer greater than or

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equal to n (derived from Eq. 14) and n^- denote the largest integer less than or equal to n. Because n^* is either n^+ or n^-, we can first treat E[$TCU(Q,n)$] (Eq. 4) as a cost function with a single-decision variable Q, and perform the following rearrangements.

*Phase 2: Searching for Q**

First, the long-run cost function E[$TCU(Q, n)$] (i.e. Eq.4) can be rearranged as the following single-decision-variable function:

$$E\left[TCU\left(Q,n\right)\right]=\alpha_1+\alpha_6\left(Q\right)+\alpha_7\left(Q^{-1}\right) \tag{15}$$

where α_6 and α_7 denote:

$$\alpha_6=\alpha_2+n^{-1}\alpha_5 \tag{16}$$

$$\alpha_7=\alpha_3+n\alpha_4 \tag{17}$$

With further rearrangements, Eq.15 becomes

$$E\left[TCU\left(Q,n\right)\right]=\alpha_1+Q\left[\sqrt{\alpha_6}-\sqrt{\alpha_7}\cdot Q^{-1}\right]^2+2\sqrt{\alpha_6}\sqrt{\alpha_7} \tag{18}$$

Upon derivation of Eq.18, it can be noted that E[$TCU(Q,n)$] will be minimised if the second term in it equals zero. That is:

$$Q^*=\sqrt{\frac{\alpha_7}{\alpha_6}} \tag{19}$$

Substituting Eq.16 and 17 into Eq. 19, the optimal production lot size is

$$Q^*=\sqrt{\frac{2\left(K+nK_1\right)\lambda}{\begin{array}{l}\left[\frac{\lambda h_1\left(E[x]\right)^2(1-\theta)^2}{P_1}+\frac{h\lambda}{P}+\frac{h\lambda}{P_1}\left[\left(2E[x]-\left(E[x]\right)^2-\varphi\left(E[x]\right)^2\right)(1-\theta)\right]\right]+\left(1-\frac{1}{n}\right)h\left(1-\varphi E[x]\right)^2\\-\left(1-\frac{1}{n}\right)\left(1-\varphi E[x]\right)\left(h-h_2\right)\left[\frac{\lambda}{P}+\frac{E[x](1-\theta)\lambda}{P_1}\right]+h_2\left(1-\varphi E[x]\right)^2\frac{1}{n}\end{array}}} \tag{20}$$

It is noted that Eq.20 is identical to that obtained using the conventional differential calculus method [13].

To find the optimal production-shipment (Q^*, n^*) policy, we substitute all related system parameters, along with n^+ and n^-, into Eq.20. Then, applying the resulting (Q, n^+) and (Q, n^-) respectively in Eq. 4, we choose the one that gives the minimum long-run average cost as the optimal production-shipment policy (Q^*, n^*). A numerical example to demonstrate the practical usage of this method is provided in the next section.

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NUMERICAL EXAMPLE

The aforementioned two-phase algebraic approach and its resulting Eq.14, 20 and 4 are verified in this section using the same numerical example [13]. Suppose an end product can be produced at a rate of 60,000 units per year, its annual demand being estimated to be 3,400 units, and during the production process there is a random defective rate x that follows a uniform distribution over a range of [0, 0.3]. A proportion θ = 0.1 of the imperfect items is considered to be scrap and the other portion is assumed to be repairable with a rework rate of P_1 = 2,100 units per year. It is further estimated that there is a proportion θ_1 = 0.1 of reworked items that fail (become scrap) during the rework period. As a quality assurance policy, the finished items can only be delivered to customers if the whole lot is quality-assured after reworking. A fixed quantity of n installments of the perfect end items are shipped to customers at a fixed interval of time during delivery time t_3 (Figure 1). Other selected parameter values in this example are as follows:

C = \$100 per item,
C_R = \$60 for each reworked item,
C_S = \$20 for each scrap item,
K = \$20,000 per production run,
h = \$20 per item per year,
h_1 = \$40 per reworked item per unit time,
K_1 = \$2,400 per shipment,
C_T = \$0.1 per item delivered,
h_2 = \$80 per item kept at the customer's end per unit time.

Applying Eq.14, we obtain n=2.736. Because the number of deliveries has to be an integer, we have n^+=3 and n^-=2. Substituting all system parameters, along with n^+ and n^- respectively, into Eq.20, we find two possible policies, namely (Q, n^+)=(1735, 3) and (Q, n^-)=(1579, 2). We then apply (Q, n^+) and (Q, n^-) in Eq.4 to obtain E[TCU(1735,3)]=\$485,541 and E[TCU(1579,2)]=\$487,071.

Selecting that with the minimum cost, we find that the optimal policy (Q^*, n^*)=(1735, 3) and the long-run average cost E[$TCU(Q^*, n^*)$]=\$485,541. The results are noted to be identical to those obtained by Chiu et al. [13].

The effect of varying the lot-size Q on the long-run average cost function E[$TCU(Q, n)$] and on the components of E[$TCU(Q, n)$], for n^* = 3, is depicted in Figure 2.

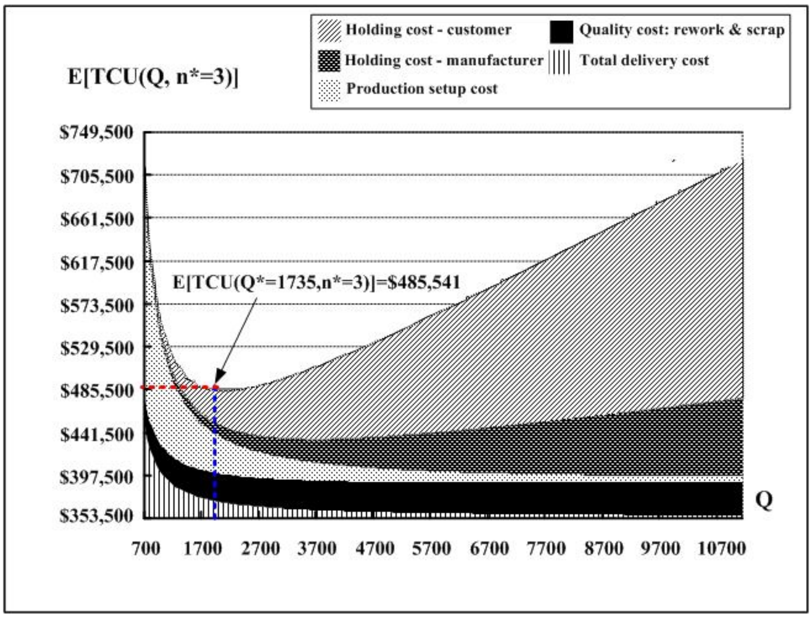

Figure 2. Effect of varying lot size Q on the long-run average cost function E[$TCU(Q, n)$] and on the components of E[$TCU(Q, n)$] for $n^* = 3$

CONCLUSIONS

This paper proposes a two-phase algebraic approach to determining the optimal production-shipment policy for an end product in an integrated supplier-customer system with quality assurance.

Unlike the conventional method, which uses differential calculus on the system cost function to find the economic lot size and optimal number of deliveries, the proposed two-phase algebraic approach is a straightforward method that may enable practitioners with little or no knowledge of differential calculus to understand and manage real-world systems more effectively. The research results were confirmed to be identical to those obtained by the traditional method

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