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Full Paper

Some properties of a subclass of non-Bazilevic functions

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Abstract: The aim of this paper is to generalise the class of non-Bazilevic functions by using the concept of differential subordinations. The inclusion relations, the coefficient bound, the covering theorem and the famous Fekete-Szego inequality related with this subclass of analytic functions are studied.

Keywords: non-Bazilevic functions, differential subordination, Fekete-Szego inequality

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INTRODUCTION

Let A denote a class of analytic functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1}$$

which is defined in the open unit disc $E = \{z : |z| < 1\}$. A function f in A is said to be a starlike function of order ρ if and only if

$$\operatorname{Re}\frac{zf'(z)}{f(z)} > \rho, \ 0 \le \rho < 1, \ z \in E.$$

This class of functions is denoted by $S^*(\rho)$. It is noted that $S^*(0) = S^*$. Let f_1 and f_2 be two functions which are analytic in E. We say that the function f_1 is subordinate to the function f_2 in E (write $f_1 \prec f_2$ or $f_1(z) \prec f_2(z)$) if there exists a function w analytic in E with $w(0) = 0$ and $|w(z)| < 1$ in E such that $f_1(z) = f_2(w(z))$. A function f in A is said to be a Janowski starlike function denoted by $S^*[A, B]$ if and only if

$$\frac{zf'(z)}{f(z)} \prec \frac{1+Az}{1+Bz}, \ -1 \le B < A \le 1, \ z \in E.$$

Obradovic [1] introduced a class of functions $f \in A$ such that

$$\operatorname{Re}\left(f'(z)\left(\frac{z}{f(z)}\right)^{\alpha}\right) > 0,\ 0 < \alpha < 1,\ z \in E.$$

This class of functions was then called a non-Bazilevic type. Tuneski and Darus [2] obtained the Fekete Szego inequality for the non-Bazilevic class of functions. Using the concept of non-Bazilevic class of functions, Wang et al. [3] studied many subordination results for the class $N(\mu, \lambda, A, B)$ defined as:

$$N(\mu, \lambda, A, B) = \left\{ f \in A : (1+\lambda)\left(\frac{z}{f(z)}\right)^{\alpha} - \lambda f'(z)\left(\frac{z}{f(z)}\right)^{\alpha+1} \prec \frac{1+Az}{1+Bz} \right\},$$

where $\lambda \in \mathbb{C}$, $-1 \le B < 1$, $A \ne B$ and $0 < \alpha < 1$.

Using the concept of subordination and Non-Bazilevicness, we generalize and define a subclass of non-Bazilevic functions as follows.

Definition 1. A function $f \in N_{\alpha,\mu}(A, B)$ if it satisfies the condition:

$$f'(z)\left(\frac{z}{f(z)}\right)^{\alpha} + \mu\left\{1 + \frac{zf''(z)}{f'(z)} - \alpha\frac{zf'(z)}{f(z)} + \alpha - 1\right\} \prec \frac{1+Az}{1+Bz},\ z \in E, \qquad (2)$$

where $\mu > 0$, $-1 \le B < A \le 1$ and $0 < \alpha < 1$.

For $A = 1-2\rho,\ B = -1$, we have the class $N_{\alpha,\mu}(\rho)$ defined as follows:

$$\operatorname{Re}\left\{f'(z)\left(\frac{z}{f(z)}\right)^{\alpha} + \mu\left(1 + \frac{zf''(z)}{f'(z)} - \alpha\frac{zf'(z)}{f(z)} + \alpha - 1\right)\right\} > \rho,\ z \in E.$$

Throughout in this paper we assume that $\mu > 0$, $-1 \le B < A \le 1$ and $0 < \alpha < 1$ unless otherwise specified.

PRELIMINARY RESULTS

We need the following lemmas, which will be used in our main results.

Lemma 1 [4]. If $-1 \le B < A \le 1, \beta > 0$ and the complex number γ satisfies $\operatorname{Re}\gamma \ge -\beta(1-A)/(1-B)$, then the differential equation,

$$q(z) + \frac{zq'(z)}{\beta q(z) + \gamma} = \frac{1+Az}{1+Bz},\ z \in E,$$

has the univalent solution in E given by

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$$q(z)=\begin{cases}\dfrac{z^{\beta+\gamma}(1+Bz)^{\beta(A-B)/B}}{\beta\int_0^z t^{\beta+\gamma-1}(1+Bt)^{\beta(A/B-1)}\,dt}-\dfrac{\gamma}{\beta}, & B\neq 0,\\[2ex] \dfrac{z^{\beta+\gamma}e^{\beta Az}}{\beta\int_0^z t^{\beta+\gamma-1}e^{\beta At}dt}-\dfrac{\gamma}{\beta}, & B=0.\end{cases}$$

If $h(z)=1+c_1z+c_2z^2+\ldots$ satisfies

$$h(z)+\frac{zh'(z)}{\beta h(z)+\gamma}\prec\frac{1+Az}{1+Bz},\ z\in E,$$

then

$$h(z)\prec q(z)\prec\frac{1+Az}{1+Bz},$$

and $q(z)$ is the best dominant.

Lemma 2 [5]**.** Let ε be a positive measure on $[0,1]$ and let g be a complex-valued function defined on $E\times[0,1]$ such that $g(.,t)$ is analytic in E for each $t\in[0,1]$ and that $g(z,.)$ is ε-integrable on $[0,1]$ for all $z\in E$. In addition, suppose that $\mathrm{Re}\{g(z,t)\}>0, g(-r,t)$ is real and $\mathrm{Re}\{1/g(z,t)\}\geq 1/(g(-r,t)$ for $|z|\leq r<1$ and $t\in[0,1]$.

If $g(z)=\int_0^1 g(z,t)d_\varepsilon(t)$, then $\mathrm{Re}\{1/g(z)\}\geq 1/g(-r)$.

Lemma 3 [6]**.** Let a, b and $c\neq 0,-1,-2\ldots$ be complex numbers. Then, for $\mathrm{Re}\,c>\mathrm{Re}\,b>0$

(i) $F(a,b,c;z)=\dfrac{\Gamma(c)}{\Gamma(c-b)\Gamma(b)}\int_0^1 t^{b-1}(1-t)^{c-b-1}(1-tz)^{-a}\,dt,$

(ii) ${}_2F_1(a,b,c;z)={}_2F_1(b,a,c;z),$

(iii) ${}_2F_1(a,b,c;z)=(1-z)^{-a}\ {}_2F_1\left(a,c-b,c;\dfrac{z}{z-1}\right).$

Lemma 4 [7]**.** Let $-1\leq B_1\leq B_2<A_2\leq A_1\leq 1$. Then

$$\frac{1+A_2z}{1+B_2z}\prec\frac{1+A_1z}{1+B_1z}.$$

Lemma 5 [8]**.** Let F be analytic and convex in E. If $f,\ g\in A$ and $f,\ g\prec F$. Then

$$\lambda f+(1-\lambda)g\prec F,\quad 0\leq\lambda\leq 1.$$

Lemma 6 [9]**.** Let $f(z)=\sum_{k=0}^{\infty}a_kz^k$ be analytic in E and $F(z)=\sum_{k=0}^{\infty}b_kz^k$ be analytic and convex in E. If $f\prec F$, then

$$|a_k|\leq|b_1|\quad(k\in\mathbb{N}).$$

Lemma 7 [10]**.** If $p(z)=1+p_1z+p_2z^2+\ldots$ is a function with positive real part in E, then

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$$|p_2 - vp_1^2| \le \begin{cases} -4v+2, & v \le 0, \\ 2, & 0 \le v \le 1, \\ 4v-2, & v \ge 1. \end{cases}$$

When $v<0$ or $v>1$, equality holds if and only if $p(z)$ is $\frac{1+z}{1-z}$ or one of its rotations. If $0<v<1$, then equality holds if and and only if $p(z)=\frac{1+z^2}{1-z^2}$ or one of its rotations. If $v=0$, equality holds if and only if $p(z)=\left(\frac{1}{2}+\frac{\eta}{2}\right)\frac{1+z}{1-z}+\left(\frac{1}{2}-\frac{\eta}{2}\right)\frac{1-z}{1+z}$, $(0\le\eta\le1)$ or one of its rotations. If $v=1$, equality holds if and only if p is the reciprocal of one of the functions such that equality holds in the case of $v=0$. Although the above upper bound is sharp, when $0<v<1$, it can be improved as follows:

$$|p_2 - vp_1^2| + v|p_1|^2 \le 2, \quad (0<v\le 1/2),$$

and

$$|p_2 - vp_1^2| + (1-v)|p_1|^2 \le 2, \quad (1/2<v\le 1).$$

Lemma 8 [11]. If $p(z)=1+p_1z+p_2z^2+\dots$ is a function with positive real part in E, then for a complex number v,

$$|p_2 - vp_1^2| \le 2\max\{1, |2v-1|\}.$$

This result is sharp for the functions

$$p(z)=\frac{1+z^2}{1-z^2}, \quad p(z)=\frac{1+z}{1-z}.$$

MAIN RESULTS

Theorem 1. If $f\in N_{\alpha,\mu}(A,B)$, then

$$f'(z)\left(\frac{z}{f(z)}\right)^{\alpha} \prec \frac{1}{1/\mu Q(z)} = q(z) \prec \frac{1+Az}{1+Bz}, \tag{3}$$

where

$$Q(z)=\begin{cases} \int_0^1 t^{\frac{1}{\mu}-1}\left(\frac{1+Btz}{1+Bz}\right)^{\frac{1}{\mu}(A-B)/B} dt \quad , & B\neq 0, \\ \int_0^1 t^{\frac{1}{\mu}-1} e^{\frac{1}{\mu}Az(t-1)} dt \quad , & B=0. \end{cases} \tag{4}$$

and $q(z)$ is the best dominant. In addition if $A<-\mu B$, $-1\le B<0$, then $N_{\alpha,\mu}(A,B)\subset N(\alpha,\rho)$, where

$$\rho=\left\{ {}_2F_1\left(1, \frac{1}{\mu}\left(1-\frac{A}{B}\right); \frac{1}{\mu}+1; \frac{B}{B-1}\right)\right\}^{-1} \tag{5}$$

This result is best possible.

Proof. Let

$$h(z) = f'(z)\left(\frac{z}{f(z)}\right)^{\alpha},$$

where $h(z)$ is analytic in E with $h(0)=1$. Differentiating logarithmically, we obtain:

$$\frac{h'(z)}{h(z)} = \frac{f''(z)}{f'(z)} - \alpha\frac{f'(z)}{f(z)} + \frac{\alpha}{z}.$$

It follows easily that:

$$h(z) + \mu\frac{h'(z)}{h(z)} = f'(z)\left(\frac{z}{f(z)}\right)^{\alpha} + \mu\left\{1 + \frac{zf''(z)}{f'(z)} - \alpha\frac{zf'(z)}{f(z)} + \alpha - 1\right\}.$$

Since $f \in N_{\alpha,\mu}(A,B)$, therefore,

$$h(z) + \frac{h'(z)}{(1/\mu)h(z)} \prec \frac{1+Az}{1+Bz}.$$

Using Lemma 1 for $\lambda = \frac{1}{\mu}$ and $\gamma = 0$, we obtain:

$$h(z) \prec \frac{1}{(1/\mu)Q(z)} = q(z) \prec \frac{1+Az}{1+Bz},$$

where $q(z)$ is the best dominant of (3) and is given by (4). Next we show that $\inf_{|z|<1}\{\operatorname{Re} q(z)\} = q(-1)$.

Now if we set $a = \frac{1}{\mu}(B-A)/B$, $b = \frac{1}{\mu}$ and $c = \frac{1}{\mu}+1$, then it is clear that $c > b > 0$. It follows from (4) for $B \neq 0$ that:

$$Q(z) = (1+Bz)^{a}\int_0^1 t^{b-1}(1+Btz)^{-a}\,dt.$$

By using Lemma 3, we get:

$$Q(z) = \frac{\Gamma(b)}{\Gamma(c)}\,{}_2F_1\left(1,a,c;\frac{Bz}{Bz+1}\right). \tag{6}$$

To prove that $\inf_{|z|<1}\{\operatorname{Re} q(z)\} = q(-1)$, we need to show that:

$$\operatorname{Re}\{1/Q(z)\} \geq 1/Q(-1).$$

Since $A < -\mu B$ with $-1 \leq B < 0$, this implies that $c > a > 0$ and it follows that:

$$Q(z) = \int_0^1 g(z,t)\,d_{\varepsilon}t,$$

where

$$g(z,t) = \frac{1+Bz}{1+(1-t)Bz},$$

$$d_{\varepsilon}t = \frac{\Gamma(c)}{\Gamma(a)\Gamma(c-a)}t^{a-1}(1-t)^{c-a-1},$$

which is a positive measure on $[0,1]$. For $-1 \le B < 0$ it is clear that $\operatorname{Re} g(z,t) > 0$ and $g(-r,t)$ is real for $0 \le |z| \le r < 1$ and $t \in [0,1]$. Now using Lemma 2, we obtain:

$$\operatorname{Re}\{1/Q(z)\} \ge 1/Q(-r).$$

Now letting $r \to 1^-$, it follows:

$$\operatorname{Re}\{1/Q(z)\} \ge 1/Q(-1).$$

Further by taking $A \to -\mu B$ for the case $A = -\mu B$ and using (3), we get $N_{\alpha,\mu}(A,B) \subset N(\alpha,\rho)$.

Theorem 2. For $\mu_2 \ge \mu_1 \ge 0$ and $-1 \le B_1 \le B_2 < A_2 \le A_1 \le 1$,

$$N_{\alpha,\mu_2}(A_2,B_2) \subset N_{\alpha,\mu_1}(A_1,B_1).$$

Proof. Let $f \in N_{\alpha,\mu_2}(A_2,B_2)$. Then

$$f'(z)\left(\frac{z}{f(z)}\right)^{\alpha} + \mu_2\left\{1+\frac{zf''(z)}{f'(z)} - \alpha\frac{zf'(z)}{f(z)} + \alpha - 1\right\} \prec \frac{1+A_2 z}{1+B_2 z},\ z \in E.$$

Since $-1 \le B_1 \le B_2 < A_2 \le A_1 \le 1$, therefore by Lemma 4, we have:

$$f'(z)\left(\frac{z}{f(z)}\right)^{\alpha} + \mu_2\left\{1+\frac{zf''(z)}{f'(z)} - \alpha\frac{zf'(z)}{f(z)} + \alpha - 1\right\} \prec \frac{1+A_1 z}{1+B_1 z},\ z \in E.$$

Hence we have $f \in N_{\alpha,\mu_2}(A_1,B_1)$. For $\mu_2 = \mu_1 \ge 0$, we have the required result. When $\mu_2 > \mu_1 \ge 0$, Theorem 1 implies that:

$$f'(z)\left(\frac{z}{f(z)}\right)^{\alpha} \prec \frac{1+A_1 z}{1+B_1 z},$$

Now

$$f'(z)\left(\frac{z}{f(z)}\right)^{\alpha} + \mu_1\left\{1+\frac{zf''(z)}{f'(z)} - \alpha\frac{zf'(z)}{f(z)} + \alpha - 1\right\}$$

$$=\left(1-\frac{\mu_1}{\mu_2}\right)f'(z)\left(\frac{z}{f(z)}\right)^{\alpha} + \frac{\mu_1}{\mu_2}\left[f'(z)\left(\frac{z}{f(z)}\right)^{\alpha} + \mu_1\left\{1+\frac{zf''(z)}{f'(z)} - \alpha\frac{zf'(z)}{f(z)} + \alpha - 1\right\}\right].$$

Using Lemma 5, we get the required result.

Theorem 3. Let $f \in N_{\alpha,\mu}(A,B)$, with $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$. Then

$$|a_2| \le \frac{A-B}{(2-\alpha)(1+\mu)}. \tag{7}$$

Proof. Since $f \in N_{\alpha,\mu}(A,B)$, therefore,

$$f'(z)\left(\frac{z}{f(z)}\right)^{\alpha} + \mu\left\{1+\frac{zf''(z)}{f'(z)} - \alpha\frac{zf'(z)}{f(z)} + \alpha - 1\right\} \prec \frac{1+Az}{1+Bz},\ z \in E.$$

Now using the fact that $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ we obtain:

$$f'(z)\left(\frac{z}{f(z)}\right)^{\alpha} + \mu\left\{1+\frac{zf''(z)}{f'(z)} - \alpha\frac{zf'(z)}{f(z)} + \alpha - 1\right\} = 1 + \{a_2(2-\alpha)(1+\mu)\}z + \ldots.$$

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By a well-known result due to Janowski and Lemma 6, we get:

$$|a_2(2-\alpha)(1+\mu)| \leq A-B.$$

$$|a_2| \leq \frac{A-B}{(2-\alpha)(1+\mu)}.$$

Therefore, we have the required result.

Theorem 4. Let $f \in N_{\alpha,\mu}(A,B)$. Then E is mapped by f on a domain that contains the disc $|w| < R_{\alpha,\mu}$, where

$$R_{\alpha,\mu} = \frac{(2-\alpha)(1-\mu)}{2(2-\alpha)(1-\mu)+(A-B)}. \tag{8}$$

Proof. Let w_0 be any complex number such that $f(z) \neq w_0$. Then

$$\frac{w_0 f(z)}{w_0 - f(z)} = z + \left(a_2 + \frac{1}{w_0}\right) z^2 + \ldots,$$

is univalent in E, so that

$$\left|a_2 + \frac{1}{w_0}\right| \leq 2$$

Therefore,

$$\left|\frac{1}{w_0}\right| - |a_2| \leq 2$$

Hence,

$$|w_0| \geq \frac{(2-\alpha)(1-\mu)}{2(2-\alpha)(1-\mu)+(A-B)} = R_{\alpha,\mu}.$$

Theorem 5. Let $f \in N_{\alpha,\mu}(A,B)$ and of the form (1). Then

$$|a_3 - ta_2^2| \leq \begin{cases} \dfrac{B_2}{2\beta}\left(2 - \dfrac{2\delta B_1 + \gamma B_2 + 2t\beta B_2}{\delta}\right), & t \leq \sigma_1, \\ \dfrac{B_2}{\beta}, & \sigma_1 \leq t \leq \sigma_2, \\ \dfrac{B_2}{2\beta}\left(-2 + \dfrac{2\delta B_1 + \gamma B_2 + 2t\beta B_2}{\delta}\right), & \sigma_2 \leq t, \end{cases}$$

where

$$\sigma_1 = -\frac{2\delta B_1 + \gamma B_2}{2\beta B_2},$$

$$\sigma_2 = \frac{4\delta - 2\delta B_1 - \gamma B_2}{2\beta B_2},$$

$$\sigma_3 = \frac{2\delta - 2\delta B_1 - \gamma B_2}{2\beta B_2},$$

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$$B_1 = 1+B,$$
$$B_2 = A-B,$$
$$\beta = (1+2\mu)(3-\alpha),$$
$$\gamma = \alpha(\alpha-3)+2\mu(\alpha-4),$$
$$\delta = (2-\alpha)^2(1+\mu)^2.$$

Further, if $\sigma_1 \le t \le \sigma_3$, then

$$|a_3 - ta_2^2| + \frac{1}{\beta B_2}\left(\frac{2\delta B_1 + \gamma B_2 + 2t\beta B_2}{2}\right)|a_2^2| \le \frac{B_2}{\beta}.$$

If $\sigma_3 \le t \le \sigma_2$, then

$$|a_3 - ta_2^2| + \frac{1}{\beta B_2}\left(\frac{4\delta - 2\delta B_1 - \gamma B_2 - 2t\beta B_2}{2}\right)|a_2^2| \le \frac{B_2}{\beta}.$$

These results are sharp.

Proof. Since $f \in N_{\alpha,\mu}(A,B)$, therefore, we have:

$$f'(z)\left(\frac{z}{f(z)}\right)^{\alpha} + \mu\left\{1+\frac{zf''(z)}{f'(z)} - \alpha\frac{zf'(z)}{f(z)} + \alpha - 1\right\} \prec \frac{1+Az}{1+Bz}.$$

Now we can get after simple calculations:

$$\begin{aligned} &f'(z)\left(\frac{z}{f(z)}\right)^{\alpha} + \mu\left\{1+\frac{zf''(z)}{f'(z)} - \alpha\frac{zf'(z)}{f(z)} + \alpha - 1\right\} \\ &= 1+(2-\alpha)(1+\mu)a_2 z + \\ &\left\{\alpha(\alpha-3)+2\mu(\alpha-4)\frac{a_2^2}{2} + (3-\alpha)(1+2\mu)a_3\right\}z^2 + \ldots \end{aligned} \tag{9}$$

Let $p(z) \prec \frac{1+Az}{1+Bz}$. Then

$$p(z) = \frac{(1-A)+(1+A)p_0(z)}{(1-B)+(1+B)p_0(z)},\ p_0 \in P,$$

where P is the well-known class of functions with positive real part. This implies that:

$$p(z) = 1+\left(\frac{A-B}{2}\right)p_1 z + \left(\frac{A-B}{2}\right)\left\{p_2 - \frac{1+B}{2}p_1^2\right\}z^2 + \ldots, \tag{10}$$

where

$$p_0(z) = 1+\sum_{n=1}^{\infty} p_n z^n.$$

From (9) and (10) after comparing the coefficients of z and z^2, we obtain:

$$a_2 = \frac{(A-B)p_1}{2(2-\alpha)(1+\mu)},$$

$$a_3 = \frac{1}{(1+2\mu)(3-\alpha)}\left[\left(\frac{A-B}{2}\right)\left\{p_2 - \frac{1+B}{2}p_1^2\right\} - \{\alpha(\alpha-3)+2\mu(\alpha-4)\}\frac{a_2^2}{2}\right].$$

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This implies that:

$$|a_3 - ta_2^2| = \frac{A-B}{2(1+2\mu)(3-\alpha)}|p_2 - vp_2^2|,$$

where

$$v = \frac{2(2-\alpha)^2(1+\mu)^2(1+B)+(A+B)\{\alpha(\alpha-3)+2\mu(\alpha-4)\}+2t(1+2\mu)(3-\alpha)(A-B)}{4(2-\alpha)^2(1+\mu)^2}.$$

Now using Lemma 7, we obtain the required result. Equality can be attained by the functions $F(z)$, defined as follows:

$$F'(z)\left(\frac{z}{F(z)}\right)^{\alpha} + \mu\left\{1+\frac{zF''(z)}{F'(z)} - \alpha\frac{zF'(z)}{F(z)} + \alpha - 1\right\} = \frac{1+Az}{1+Bz}, \text{ if } t < \sigma_1, \text{ or } t > \sigma_2,$$

$$F'(z)\left(\frac{z}{F(z)}\right)^{\alpha} + \mu\left\{1+\frac{zF''(z)}{F'(z)} - \alpha\frac{zF'(z)}{F(z)} + \alpha - 1\right\} = \frac{1+Az^2}{1+Bz^2}, \text{ if } \sigma_1 < t < \sigma_2,$$

$$F'(z)\left(\frac{z}{F(z)}\right)^{\alpha} + \mu\left\{1+\frac{zF''(z)}{F'(z)} - \alpha\frac{zF'(z)}{F(z)} + \alpha - 1\right\} = \frac{1+A\phi(z)}{1+B\phi(z)}, \text{ if } t = \sigma_1,$$

and

$$F'(z)\left(\frac{z}{F(z)}\right)^{\alpha} + \mu\left\{1+\frac{zF''(z)}{F'(z)} - \alpha\frac{zF'(z)}{F(z)} + \alpha - 1\right\} = \frac{1-A\phi(z)}{1-B\phi(z)}, \text{ if } t = \sigma_2,$$

where $\phi(z) = \frac{z(z+\eta)}{1+\eta z}$ with $0 \le \eta \le 1$.

Theorem 6. Let $f \in N_{\alpha,\mu}(A,B)$ and of the form (1). Then for a complex number t,

$$|a_3 - ta_2^2| \le \frac{B_2}{\beta}\max\left\{1, \left|-\frac{2\delta B_1 + \gamma B_2 + 2t\beta B_2}{2\delta} + 1\right|\right\}.$$

By using Lemma 8, we have the required result. Equality can be attained by the function:

$$F'(z)\left(\frac{z}{F(z)}\right)^{\alpha} + \mu\left\{1+\frac{zF''(z)}{F'(z)} - \alpha\frac{zF'(z)}{F(z)} + \alpha - 1\right\} = \frac{1+Az}{1+Bz},$$

or

$$F'(z)\left(\frac{z}{F(z)}\right)^{\alpha} + \mu\left\{1+\frac{zF''(z)}{F'(z)} - \alpha\frac{zF'(z)}{F(z)} + \alpha - 1\right\} = \frac{1+Az^2}{1+Bz^2}.$$

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